MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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F. W. DYSON, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Arthur Gilbert Fellows, 18 Wellington Road, Watford, Herts;
Charles Whitsey Franklin, 15 The Grove, Swansea, South Wales;
David Llewellyn Howe, 12 Leigh Road, Clifton, Bristol; and
James Mason, M.A., B.Sc., Sutherland Technical School, Golspie, Scotland,

were balloted for and duly elected Fellows of the Society.

Baron Roland Eötvös, Professor of Physics in the University, Budapest, Hungary; and
Annibale Riccò, Director of the Observatory, Catania, Sicily,

were balloted for and duly elected Associates of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Sarat Chandra Bhattacharya, M.A., F.C.S., F.R.Met.Soc., Professor, St. Columba's College, Hazaribagh, Bengal, India (proposed by Jnan Saran Chakravarti);
Rev. Martin Davidson, B.A., B.Sc., 56 Hudson's Road, Canning Town, E. (proposed by F. W. Levander);
Philip Fox, Director of the Dearborn Observatory, Evanston, Illinois, U.S.A. (proposed by S. W. Burnham);
John Rudd Leeson, M.D. Edin., F.L.S., F.G.S., Clifden House,

Twickenham, and Athenæum Club, S.W. (proposed by F. W. Dyson); and

Hugh Wright, Librarian, Mitchell Library, Sydney, New South Wales, Australia (proposed by John Tebbutt and R. T. A. Innes).

Seventy-five presents were announced as having been received since the last meeting, including, amongst others:—

Bombay Government Observatory, Magnetic Observations for the Period 1846 to 1905, presented by the Observatory; Oxford University, Astrographic Catalogue, Oxford Section, vol. 7; and Tables for Conversion of R.A. and Dec. into Standard Co-ordinates, etc., presented by the Observatory; E. T. Whittaker, A History of the Theories of Aether and Electricity, presented by the Author.

Five lantern slides from photographs of Halley's Comet, taken at the Helwân Observatory, 1910 May 30 to June 4, presented by W. Knox Shaw.

An 8-inch Cooke equatorial refractor which had belonged to the late Mr. W. Coleman, and had been presented to the Society by the Executors in accordance with the terms of his Will.

The Analytical Solution of the Problem of Orbits.

By C. V. L. Charlier, Assoc. R.A.S.

1. In three memoirs published during the years 1778–83 Lagrange has given the general solution of the problem, to determine the orbit of a comet or a planet from three complete observations. Singularly enough, where such an eminent author is concerned, these researches have fallen into an almost complete oblivion, all the numerous applications of the problem of orbits of the preceding century being based on researches published by other astronomers some twenty years after the appearance of the memoirs of Lagrange.

I happened to read these memoirs of Lagrange at the end of the last year, and it struck me at once that the solution of Lagrange was not only a general and complete solution of the problem of orbits, but that it was possible to get in this way a purely analytical solution of the problem in the form of power series proceeding according to the ascending powers of the time-intervals between the three observations; a solution where any sort of successive approximations is entirely avoided, and where the calculation of the elements can be carried to any degree of accuracy compatible with the errors in the observations. Consequently I proposed to elaborate such a solution. The result will be found in the next "Meddelanden" from the observatory in Lund. I will here give a short *resumé* of the method.

2. The essential feature of the method of Lagrange is, that the calculation of the elements is based on the values of three quantities, p, q, and u, defined in the following manner:—

$$u = \frac{1}{r^3},$$

$$p = \frac{1}{2r^2}\frac{dr^2}{dt} = \frac{1}{r^2}\left(x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}\right), \tag{1}$$

$$q = \frac{1}{2r^2}\frac{d^2r^2}{dt^2} = \frac{1}{r^2}\left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right\} - u,$$

where x, y, z denote the heliocentric coordinates of the planet, and $r^2 = x^2 + y^2 + z^2$.

The quantities p, q, and u being known, the elements of the orbit are obtained through simple formulæ. The major axis of the orbit is found from the formula

$$\frac{1}{a} = \frac{1}{r} - r^2q,$$

the semi-parameter [$= a(1 - e^2)$ or b] from

$$b = r + r^4(q - p^2).$$

The angle v between the radius vector and the perihelion is found from

$$e \cos v = r^3(q - p^2).$$

The formulæ for the other elements it is unnecessary here to reproduce.

3. The reason why the quantities p, q, and u are especially suitable for use in the problem of orbits is the very simple manner in which these quantities occur in the development of the coordinates in powers of the time. Let x, y, z denote the values of the heliocentric coordinates at any time t; then we have

$$x = fx_0 + g\frac{dx_0}{dt},$$

$$y = fy_0 + g\frac{dy_0}{dt}, \tag{2}$$

$$z = fz_0 + g\frac{dz_0}{dt},$$

x_0, y_0, z_0 denoting the values of the coordinates at a given time t_0. In these expressions the functions f and g denote certain series proceeding according to ascending powers of $t - t_0$. *The coefficients in these series are polynomials in the quantities p, q, and u.*

Owing to this simple fact, the final equations for determining p, q, and u in the problem of orbits have the form

$$\begin{aligned} &F(p, q, u, t_1 - t_0, t_2 - t_0) = 0, \\ &G(p, q, u, t_1 - t_0, t_2 - t_0) = 0, \\ &H(p, q, u, t_1 - t_0, t_2 - t_0) = 0, \end{aligned} \tag{3}$$

t_0, t_1, t_2 being the epochs of the three observations, and F, G, and H being three functions developable in powers of $t_1 - t_0$ and $t_2 - t_0$. The coefficients of the different powers of the time-intervals are here polynomials in p, q, and u—with the sole exception of the term independent of t—* and by the inversion of these series p, q, and u are expressed in the desired form of power series in $t_1 - t_0$ and $t_2 - t_0$.

4. The equations (3) can be formed in the following manner.

Using, for instance, the ratio of the triangles, as in the method of Olbers and Gauss, or proceeding as in the more direct method of Lagrange, the distances ρ, ρ_1, and ρ_2 at the epochs t_0, t_1, and t_2 may be expressed in the form

$$\begin{aligned} \rho &= Es + Fv + Gw, \\ \rho_1 &= E_1 s_1 + F_1 v_1 + G_1 w_1, \\ \rho_2 &= E_2 s_2 + F_2 v_2 + G_2 w_2, \end{aligned} \tag{4}$$

where E, F, G, E_1, etc., are certain coefficients known from the observed coordinates of the planet, whereas s, v, w, s, etc., denote certain power series in the time-intervals with coefficients that are simple functions of p, q, and u.

From the triangle Earth-planet-Sun we get, however, for the three epochs, the relations

$$\begin{aligned} \rho^2 + 2m\rho + R^2 &= r^2, \\ \rho_1^2 + 2m_1\rho_1 + R_1^2 &= r_1^2, \\ \rho_2^2 + 2m_2\rho_2 + R_2^2 &= r_2^2, \end{aligned} \tag{5}$$

where m, m_1, and m_2 are known functions of the observed coordinates. We have, moreover,

$$\begin{aligned} r_1^2 &= h_1 r^2, \\ r_2^2 &= h_2 r^2, \end{aligned}$$

where h_1 and h_2 are power series in the time-intervals $t_1 - t_0$ and $t_2 - t_0$ respectively, with coefficients dependent only on p, q, and u.

Substituting now the three series (4) in (5), we have immediately the relations (3), from which p, q, and u can be found as power series in $t_1 - t_0$ and $t_2 - t_0$.

I have formed these relations and calculated the two first terms

* Which gives the equation of Lagrange, dealt with in my previous notes to the R. A. S.

in these series. Applying the result to numerical examples, I have found complete agreement with other methods for the calculation of orbits.

5. In the usual formulæ for the distances of a planet or a comet from the Earth, corresponding to the formulæ (4) above, there occur also negative powers of the time-intervals (of the first degree). Evidently the nature of the problem demands that these negative powers must vanish. This result is, indeed, easily reached through the introduction of certain *difference-coefficients*, regarding which I refer to the "Meddelanden."

6. The representation of the solution in the form of power-series in $t_1 - t_0$ and $t_2 - t_0$ is impossible if the planet (or comet) from the first to the third observation passes a certain surface, which I have called the singular surface of the problem of orbits.* The desired quantities are then expressible through the root of a certain quadratic

$$x^2 + P_1 x + P_2 = 0,$$

where P_1 and P_2 denote power series in the time-intervals $t_1 - t_0$ and $t_2 - t_0$.

The form of the singular surface is seen from my last communication to the R.A.S.

7. The value of the radius of convergence of the power series, representing the elements, is discussed, though not finally determined, in the "Meddelanden." Researches of Moulton and W. A. Hamilton on the development of the coordinates in the problem of two bodies are here of fundamental importance. It remains, however, to discuss the influence of the singular surface on the value of the radius of convergence.

8. The contributions of the parallax and of the aberration to the values of the elements are very easy to take account of analytically. Thus in the analytical method the elements of the orbit of a planet or a comet are obtained in the most simple manner mathematically, and simultaneously, I think, in the most simple manner for numerical work.

* I suppose, though I have not examined the question more closely, that even the usual methods for calculation of orbits—being implicitly based on a development in powers of the time-intervals—fail under these circumstances.

Lund:
1911 *June* 4.

Further considerations relating to the systematic motions of the Stars. By J. Halm, Ph.D.

(*Communicated by H.M. Astronomer, Cape of Good Hope.*)

In a paper entitled "On the velocity of the Sun's motion through space as derived from the radial velocity of Orion Stars," which appeared in vol. xxxii., No. 1, of the *Astrophysical Journal*, Messrs. Kapteyn and Frost point out some important anomalies in the radial motions of these stars, which appear to be intimately connected with similar results previously derived from a discussion of the radial motions of stars of more advanced types (*Monthly Notices*, vol. lxx., November 1909).

A more direct comparison of their results with those obtained by Mr. Hough and myself seemed therefore desirable in order to define more precisely the *rôles* played by these two groups of stars in the motions of the universe, and to point out both the common and diverse features of their motions.

This investigation was practically completed at the time of the publication of Professor Boss's Provisional General Catalogue. It was at once recognised that an important opportunity had arrived for testing some of the conclusions derived from the examination of the radial velocities by means of the transverse motions supplied by this exhaustive and reliable source of information. The principal question on which information from the transverse motions was desired referred to the particular features characterising the Orion type stars as distinguished from the others. An examination of the proper motions of the Boss Stars was well in hand when Mr. Eddington's paper appeared in the *Monthly Notices*, which to a large extent supplied the answer to this question. But on closer examination it was found that those particular features which distinguish the Orion stars from the others are shared by a much larger number of stars than has so far been assumed. The evidence on which this conclusion is based will form a special chapter of the present investigation.

A further point examined in this paper is the question of inequality of distribution of drift, and its bearing on the precessional constant derived from the stars of the Boss Catalogue.

Finally, some of the prominent facts of star motion are discussed from the point of view of the Maxwellian law of distribution, which establishes a necessary relation between the average masses and peculiar motions of the stars.

Examination of Radial Velocities.—A discussion of the radial motions of 491 brighter stars, *not* including those of the Orion type, was published in *M.N.*, vol. lxx., November 1909. It was found that, in addition to the motion of these stars which could be ascribed to a uniform relative motion with reference to the Sun, and which therefore analytically should be expressible by first order spherical harmonics, there was strong evidence of the existence of second

order harmonics, which could be qualitatively accounted for by the assumption of an inequality in the mixture of the two drifts in various parts of the sky.

The examination of this point has now been taken up with special reference to those stars of our original list which are situated between $\pm 30^\circ$ galactic latitude, the reason for this selection being that the Orion stars with which our results are to be compared show a marked preference for crowding in the vicinity of the Milky Way. After subtracting the solar motion represented by the numerical formula found in the paper (*M.N.*, vol. lxx., p. 99), viz.,

$$v_\odot = -0\cdot4 \cos\alpha \cos\delta + 18\cdot8 \sin\alpha \cos\delta - 9\cdot0 \sin\delta \text{ km/s} \quad . \quad (1)$$

the residuals were arranged in nine groups along the galactic circle, and within each group means were taken according to the weights of the individual data. The resulting mean group residuals are shown in the third vertical column of Table I.

Table I.

Mean R.A.	Mean Decl.	Mean Residuals.	Corrected Residuals.	Formula (3).	No. of Stars.
h	°	km.			
2·1	+48	+ 2·2	−0·6	−1·3	19
5·0	+16	+ 9·7	+4·7	+5·7	25
6·6	−10	+10·7	+6·1	+7·1	24
8·2	−30	+ 3·1	+0·9	+2·0	26
10·8	−56	− 1·6	−1·1	−1·1	13
16·1	−46	− 3·9	+0·5	+3·6	13
18·3	− 6	+ 2·8	+8·1	+7·1	29
20·1	+16	+ 2·4	+6·5	+5·5	31
22·8	+49	− 0·1	−0·5	−1·3	8

It is at sight evident that these residuals are of a highly systematic character, and the adopted solar motion quite insufficient to account for the observed motions. The first question is how far they can be reduced by changing the constants of equation (1), *i.e.* by altering the amount and direction of the solar motion. An analysis of the numerical data shows that corrections in the first order terms amounting to

$$-1\cdot3 \cos\alpha \cos\delta + 5\cdot0 \sin\alpha \cos\delta + 2\cdot5 \sin\delta$$

are required. The solar motion satisfying the galactic stars would therefore be

$$v_\odot = -1\cdot7 \cos\alpha \cos\delta + 23\cdot8 \sin\alpha \cos\delta - 6\cdot5 \sin\delta \quad . \quad (2)$$

Subtracting the corrections to solar motion from the residuals of

the third column, the "corrected residuals" of the fourth column are obtained. They show a clearly pronounced double periodicity, which seems to be satisfied by a term of the form

$$2{\cdot}1 - 4{\cdot}8 \cos 2a \cos^2 \delta - 1{\cdot}3 \sin 2a \cos^2 \delta \quad . \quad . \quad (3)$$

the numerical values of which are entered in the fifth column of Table I.

Now let us compare these results with those derived by Kapteyn and Frost for the Orion type stars. Their investigation is limited to stars within moderate distances from the apex and antapex of the solar motion. It therefore does not lend itself to a complete analysis over the whole galactic circle, and all that could be achieved was a determination of the velocity of the Orion stars relatively to these two points. The solar apex assumed in Kapteyn and Frost's paper is $A = 269^\circ{\cdot}7$ and $D = +30^\circ{\cdot}8$. The principal result arrived at is the fact that the solar motion at the apex is 10 km. less than that near the antapex. To quote their figures, we have

$$\text{Orion Stars} \begin{cases} \text{at apex} & v = -18{\cdot}4 \pm 1{\cdot}4 \text{ km.} \\ \text{at antapex} & v = +28{\cdot}4 \pm 1{\cdot}4 \quad ,, \end{cases}$$

—*i.e.* the Orion stars are seen to approach our system at the apex at the rate of 18·4 km. per sec., and to recede from us at the antapex at the rate of 28·4 km. per sec.

On computing the corresponding velocities of the galactic stars as they would follow from the combined formulæ (2) and (3), we find

$$\text{Galactic stars in general} \begin{cases} \text{at apex} & v = -17{\cdot}2. \\ \text{at antapex} & v = +30{\cdot}4. \end{cases}$$

The agreement being as close as we can reasonably expect, we may safely say that, with regard to the anomaly here under consideration, there is no striking difference between the behaviour of the galactic stars in general and the Orion type stars examined by Kapteyn and Frost. The phenomenon which they were able to trace is the local expression at the apices of the more general anomaly to which attention was drawn in *M.N.*, vol. lxx., November 1909, and which was there attributed to unequal mixture of the two drifts. They are indeed inclined to ascribe their results to such a cause, but fail to see the correctness of the above formula, which they maintain "gives an identical mixture for any two points of the sphere diametrically opposite." This, we need scarcely say, is an erroneous conclusion drawn from the analysis, which is based on the exactly opposite assumption, viz. that the two drifts are *unequally* mixed on opposite points of the sphere. The only way to explain the error into which they have fallen is perhaps to be found in the fact that they assign the negative sign to the motions of the stars both at apex and antapex (see (4), p. 87 of their paper),

whereas in reality the one is a motion of recession and the other of approach, and they are therefore of opposite signs. The second order terms, introduced by unequal mixture, which certainly are equal in amount and sign at opposite points, must therefore tend to reduce the absolute amount of the velocities on the one side and increase it on the other.

The general result so far obtained from the examination of the galactic stars points to the conclusion that the systematic motions of these stars are satisfactorily expressed by the formulæ (2) + (3), viz.,

$$V = -1\cdot7 \cos\alpha\cos\delta + 23\cdot8 \sin\alpha\cos\delta - 6\cdot5 \sin\delta + 2\cdot1 \\ - 4\cdot8 \cos 2\alpha \cos^2\delta - 1\cdot3 \sin 2\alpha \cos^2\delta \quad (4)$$

Comparing the terms of this equation with the corresponding terms derived from the stars distributed over the whole sphere, viz.,

$$-0\cdot4 \cos\alpha\cos\delta + 18\cdot8 \sin\alpha\cos\delta - 9\cdot0 \sin\delta + 2\cdot1 \\ - 2\cdot2 \cos 2\alpha \cos^2\delta - 2\cdot5 \sin 2\alpha \cos^2\delta \quad . \quad (5)$$

we may perhaps hesitate to accept some of the differences as purely accidental. Considering that the galactic zone contains 2/5 of the total number of stars, it would follow that the motions of the extra-galactic stars are represented by the formula

$$V = +0\cdot5 \cos\alpha\cos\delta + 15\cdot5 \sin\alpha\cos\delta - 10\cdot8 \sin\delta + 2\cdot1 \\ - 0\cdot5 \cos 2\alpha \cos^2\delta - 3\cdot3 \sin 2\alpha \cos^2\delta \quad (6)$$

The mean solar motion derived from the first three terms of equations (4) and (6) would be

	For *galactic stars :*	For *extra-galactic stars :*
	$V = 24\cdot7 \pm 1\cdot5$ km/s.	$V = 18\cdot9 \pm 1\cdot3$ km/s.
Apex	$A = 274^\circ$	$A = 268^\circ$.
	$D = +15^\circ$	$D = +35^\circ$.

These differences in the amount and direction of the solar motion are worthy of attention. It will be noticed that the apex found from extra-galactic stars is in good agreement with that derived by Boss from the transverse motions, viz. $A = 270^\circ\cdot5$ $D = +34^\circ\cdot3$; but that of the galactic stars is considerably displaced towards the apparent apex of the first drift. The direction of the displacement and the increased velocity would jointly point to the conclusion that the first drift stars preponderate in the galaxy, a result which is at variance with that obtained in the discussion of the Bradley stars (*M.N.*, vol. lxx., June 1910), where the opposite conclusion was arrived at, and also with that of Eddington, whose

results, derived from the proper motions of the Boss Catalogue, point to a uniformity of mixture inside and outside the galaxy. An explanation of this remarkable divergence of conclusions will be given later on.

Second Order Terms.—The conclusive evidence of the existence of these terms expressed by formula (3), and shown in their effect on the observed motions of the galactic stars in the 4th column of Table I., may be considered the most important result of this part of the investigation. It is obvious that residuals exhibiting such a marked regularity and an amplitude of about 1/4 of the amplitude of the solar motion cannot be passed over without the closest scrutiny.

It will be remembered that the same terms were previously evidenced (*M.N.*, vol. lxx., November 1909) when the observing material was arranged symmetrically with reference to the equator. Taking the means of the values of Table IV. of that paper for the northern and southern hemisphere, in order to eliminate the terms multiplied into sin 2δ, we obtain the following table of residuals arranged in order of right ascension :—

TABLE II.

h h	km.	h h	km.	h h	km.	h h	km.
0–2	+0·4	6– 8	+7·5	12–14	−0·2	18–20	+5·8
2–4	−3·5	8–10	+2·7	14–16	−1·8	20–22	+4·2
4–6	+3·8	10–12	+0·5	16–18	+2·8	22– 0	+3·5

Further, it was mentioned in paper (*M.N.*, vol. lxx., June 1910) that Newcomb had found terms of the same character in his discussion of the precession derived from the transverse motions of the Bradley stars. The appearance of Boss's General Catalogue and the detailed discussion of the proper motions in various papers in the *Astronomical Journal* by Professor Boss offered an opportunity for testing the validity of Newcomb's result. The data for this examination are to be found in Table I. of Professor Boss's first paper on "Precession and Solar Motion" in *Astron. Journ.*, vol. xxvi., No. 12, p. 97. In the table referred to, the mean components of the proper motions $\mu_\alpha \cos\delta$ and μ_δ are given in order of R.A. for trapeziums of equal area arranged in zones parallel to the equator with centres at 0°, ±20°, ±40°, ±60°, and ±80°. In conformity with the arrangement of Table II. of the present paper, the observed $\mu_\alpha \cos\delta$ shown in the 7th column of Professor Boss's table of the trapeziums lying between 0^h–2^h, 2^h–4^h, . . . 22^h–0^h and ±50° Decl., and again independently of the trapeziums lying between 1^h–3^h, 3^h–5^h, . . . 23^h–1^h and the same range of declination, have been combined. As the terms in question are multiplied by cos δ, the trapeziums near the poles may be left out of consideration.

The mean group values $\mu_\alpha \cos\delta$ thus obtained were then analysed with regard to the terms involving the component of the

Sun's motion on the assumption of a random distribution of the peculiar star motions, *i.e.* the terms $-V_1 L\varpi \sin\alpha + V_1 M\varpi \cos\alpha$ of equation (1) on p. 585 of the paper, *M.N.*, vol. lxx., June 1910. The residuals obtained after subtracting these quantities are contained in the 2nd column of Table III.

TABLE III.

Mean R.A.	Residuals.	Smoothed.	Mean R.A.	Residuals.	Smoothed.
h	″	″	h	″	″
0·1	−0·91	−0·76	12·1	−0·37	−0·43
0·9	−0·67	−0·78	13·1	−0·38	−0·48
2·2	−0·75	−0·60	14·2	−0·69	−0·57
2·8	−0·37	−0·39	14·8	−0·63	−0·41
4·1	−0·05	−0·15	16·1	+0·10	−0·11
5·0	−0·02	−0·04	16·8	+0·19	−0·01
6·0	−0·05	−0·10	18·0	−0·33	−0·11
7·2	−0·24	−0·18	19·0	−0·18	−0·19
8·1	−0·20	−0·29	20·0	−0·06	−0·22
9·3	−0·43	−0·36	21·1	−0·43	−0·28
9·9	−0·46	−0·48	21·9	−0·36	−0·50
11·2	−0·56	−0·46	22·8	−0·70	−0·56

The "smoothed" residuals of the 3rd column are obtained in the usual way by combining three neighbouring values into one.

Fig. 1 represents the graphs of the residuals of Tables II. and III. They present a striking similarity to one another, and fulfil very nearly the condition demanded by theory that the phase of the transverse motions should precede that of the radial motions by three hours. One peculiarity exhibited by both curves is deserving of special attention, viz. the steep and quick ascent from a minimum to the following maximum, and the slow and protracted descent to the following minimum.

To explain this, let us revert to the numerical expressions for the second order terms (including the constant) shown in equations (4) and (6) for galactic and extra-galactic regions. We found

$$+2\cdot1 - 4\cdot8 \cos 2\alpha \cos^2\delta - 1\cdot3 \sin 2\alpha \cos^2\delta \text{ for galactic regions,}$$
$$+2\cdot1 - 0\cdot5 \cos 2\alpha \cos^2\delta - 3\cdot3 \sin 2\alpha \cos^2\delta \text{ for extra-galactic regions,} \quad (7)$$

Now the galaxy intersects the equator at about 7^h and 19^h right ascension at an angle of approximately 67°. Roughly speaking, the stars between ±50° declination and R.A. 5^h–19^h and 17^h–21^h may be considered as predominantly galactic, and *vice versa*, the stars between R.A. 22^h–4^h and 10^h–16^h as predominantly extra-galactic.

Computing now the curve of the second order terms for the intervals 5^h–9^h and 17^h–21^h from the first of the expressions (7), we find two maxima at $6^h{\cdot}5$ and $18^h{\cdot}5$, while the curve for the intervals 22^h–4^h and 10^h–16^h shows maxima at $2^h{\cdot}7$ and $14^h{\cdot}7$. The interval between a minimum and following maximum is therefore $3^h{\cdot}8$, and between a maximum and following minimum $8^h{\cdot}2$, in close agreement with the evidence of the curves. We mention these details only to emphasise the very close resemblance of two sets of systematic residuals derived from entirely different material. The physical cause of the phenomenon will be demonstrated at a later stage of this paper. In the meantime let it be

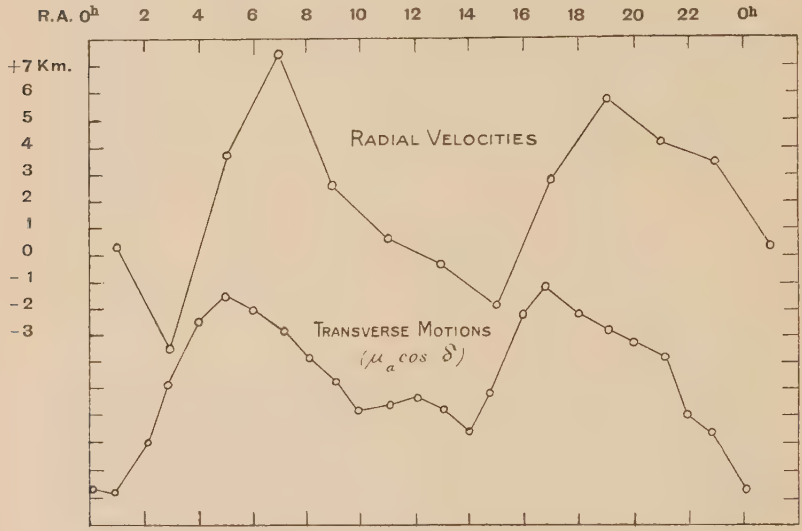

FIG. 1.—Residuals after eliminating Sun's Motion. (Stars grouped symmetrically to Equator.)

stated that the existence of such residuals, whatever their cause may be, is strongly opposed to the view of a random distribution of stellar proper motions.

Evidence of two Drifts from Radial Velocities.—In paper (*M.N.*, vol. lxx., November 1909), it was shown that the second order terms appearing in the radial velocities could be taken as evidence of two star drifts, which are mixed in unequal proportions in opposite parts of the sky. The investigation based on this assumption led to a determination of the true vertex of the two drifts in good accordance with that found by other and more direct methods. An alternative method of testing the two-drift theory by radial motions was suggested by Kapteyn, and applied by him with success in an examination of a limited number of bright stars. Its principle may be briefly stated as follows:—

Let V denote the observed radial velocity. This quantity, apart from accidental errors of measurement which may be neglected, is made up of (*a*) the radial component of the peculiar velocity of the star, ω, and the radial component of the systematic motion of the star relatively to our system, $v_\odot$; *i.e.*,

$$V = \omega + v_\odot,$$

where ω has the characteristics of an "accidental" error. Now, in the case of two drifts, where a star selected at random may belong to either of the two, we shall have either $V_1 = \omega + v_1$ or $V_2 = \omega + v_2$, where v_1, v_2, denote the radial components of the apparent systematic drift motion of Drifts I. and II.

If, among the number of stars contained in a certain area, there are s_1 stars belonging to the first drift and s_2 stars belonging to the second drift, we combine

$$s_1 V_1 + s_2 V_2 = (s_1 + s_2)\omega + s_1 v_1 + s_2 v_2,$$

and the resultant radial velocity which is obtained by dividing by the total number of stars

$$V = \omega + \frac{s_1}{s_1 + s_2} v_1 + \frac{s_2}{s_1 + s_2} v_2,$$

which, by assuming the total number of stars equal to unity, takes the form

$$V = \omega + v_1 + s_2(v_2 - v_1) \quad . \quad . \quad . \quad (8)$$

Now it is clear that the terms $[v_1 + s_2(v_2 - v_1)]$ represent the radial component of what is usually called the Sun's motion, *i.e.* the motion of our system relatively to all the stars combined. We therefore write

$$v_\odot = v_1 + s_2(v_2 - v_1),$$

and subtracting this quantity from the observed velocities of each star we obtain

$$\begin{aligned} &\text{for Drift I. stars: } (V_1 - v_\odot) = \omega - s_2(v_2 - v_1) \\ &\text{for Drift II. stars: } (V_2 - v_\odot) = \omega + (1 - s_2)(v_2 - v_1) \end{aligned} \quad (9)$$

and hence in the mean of all stars of Drifts I. and II.,

$$\begin{aligned} (V_1 - v_\odot)^2 &= \omega^2 + s_2^2(v_2 - v_1)^2 \\ (V_2 - v_\odot)^2 &= \omega^2 + (1 - s_2)^2(v_2 - v_1)^2. \end{aligned}$$

Multiplying the first equation by its weight $(1 - s_2)$, the second by

s_2, and adding, we obtain the square of the residual of unit weight

$$V^2 = \omega^2 + s_2(1 - s_2)(v_2 - v_1)^2 \quad . \quad . \quad . \quad (10)$$

We have now still to consider that $(v_2 - v_1)$ expresses the radial component of the motion of the one drift with regard to the other. Its direction is indicated by the true vertex; and if we call v the relative velocity of the two drifts in that direction, we have the relation

$$(v_2 - v_1) = v \cos \lambda \, * \quad . \quad . \quad . \quad (11)$$

where λ represents the angular distance between the vertex and the centre of the area under consideration. Substituting (11) into (10) we finally get

$$V^2 = \omega^2 + s(1 - s_2)v^2 \cos^2 \lambda \quad . \quad . \quad . \quad (12)$$

The meaning of this formula is obvious. If we subtract from the observed radial velocity of each individual star the radial component of the Sun's motion and square the residuals, these squares must attain maxima values at two opposite points of the sphere, viz. the vertex and antivertex, in case the stars move in two streams. They will be evenly distributed either for $s_2 = 0$, or $s_2 = 1$, *i.e.* when only one of the drifts exists.

We have attempted to examine from this point of view the galactic stars which are particularly suitable as the vertex appears to be situated in the Milky Way. Unfortunately, however, the material is not advantageous for another reason. The velocities are not given individually for each star, but on the average two or three neighbouring stars have been combined into groups. The chances are, therefore, that stars belonging to different drifts have been united into one and the same group. This, no doubt, would tend to diminish the effect of the second term of equation (12). The separation into individual stars would have been possible for those of the Cape list, by far the smaller number; but no data being available for the northern stars, we had to be satisfied with the existing material. Of course, if we find a pronounced accumulation of errors near the vertices even under these unfavourable conditions, the existence of two drifts would be proven *a fortiori*.

The result of this examination is shown in Table IV. A double calculation has been made, in the first case by neglecting the second order terms, and in the second case by taking them into account. The tabulated quantities represent the residual mean errors of the radial velocity of a single star, as determined from the weighted groups, after elimination of the systematic motion.

* The quantity v is twice the quantity which Kapteyn defines as the "cloud velocity."

Table IV.

Mean R.A.	Mean Decl.	Mean Error of Single Star: (Second Order Terms Excluded.)	(Second Order Terms Included.)
h	°	km.	km.
2·1	+48	±14·8	±12·3
5·0	+16	22·6	15·3
6·6	−10	29·0	22·6
8·2	−30	23·4	22·9
10·8	−56	13·1	13·9
16·1	−46	16·9	13·9
18·3	− 6	25·8	24·0
20·1	+16	23·4	23·4
22·8	+49	15·8	15·7

The figures show distinct accumulations of the residual errors at two opposite points of the galactic circle, approximately where the galaxy intersects the equator. The result would therefore prove that the motions of the stars are not reconcilable with the assumption of a single drift, but point to the existence of two star streams, a result in agreement with that found by Mr. Hough and myself in our first paper, and with the conclusions arrived at by Kapteyn from the discussion of a more limited number of bright stars.

The error due to peculiar motion of the star is represented by the minimum value in the above table. It amounts to about ±15 km. in the case where second order terms are excluded, and this value may be adopted as more comparable with similar values derived by Kapteyn. This being the "mean" error, the average error is $\frac{15}{1{\cdot}25} = \pm 12$ km., which is again in good agreement with the corresponding values for the same class of stars obtained by Kapteyn and Campbell.

The above results are in marked contrast with the conclusions at which Kapteyn and Frost have arrived regarding the motions of the stars of the Orion type. Not only have they found that the average peculiar speed of these stars amounts to only ±6·3 km., *i.e.* about half the amount shown by the other stars, but they have conclusively shown that the cloud velocity of these stars is practically nil. The Orion type stars appear to form a *single* drift, the motion of which is directed neither towards the apparent apex of the first nor towards that of the second drift, but towards the apparent apex of the two drifts as a whole, *i.e.* towards the solar antapex.

Statistical evidence of drift motion derived from the Stars of Professor Boss's General Catalogue.

The extraordinary and exceptional position of the Orion type stars appeared of such vital importance for the progress of our knowledge of the systematic motions of the universe that the

publication of Professor Boss's General Catalogue was anxiously awaited in order to examine the transverse motions of this class of stars in relation to other spectral types. Unfortunately the work, which was taken in hand on arrival of Professor Boss's publication, was unavoidably delayed, and in the meantime Eddington's valuable paper "On the systematic motions of the Stars in Professor Boss's Provisional General Catalogue" appeared in the *Monthly Notices*, in which, partly at least, the answer which we searched for was given.

Eddington's researches seem to confirm Kapteyn and Frost's conclusions that the Orion stars form a drift by themselves, moving in the direction of the solar antapex.

But the most noteworthy result of his investigation is that, even after eliminating the Orion stars, the numbers show distinct tendencies to accumulate in this particular direction, tendencies shown in his diagrams by a marked bulging of the curves.

Naturally this phenomenon did not escape his attention, and he accounts for it—justly, we think—by the presence of stars which, though not of the Orion type, nevertheless share in that peculiar motion by which the former are characterised. Where we differ from him, however, is in the estimate of their numbers. Eddington asserts that they are but few, whereas the outcome of the present investigation ascribes to them numbers considerably in excess of those of the "principal" two drifts. The point which Eddington

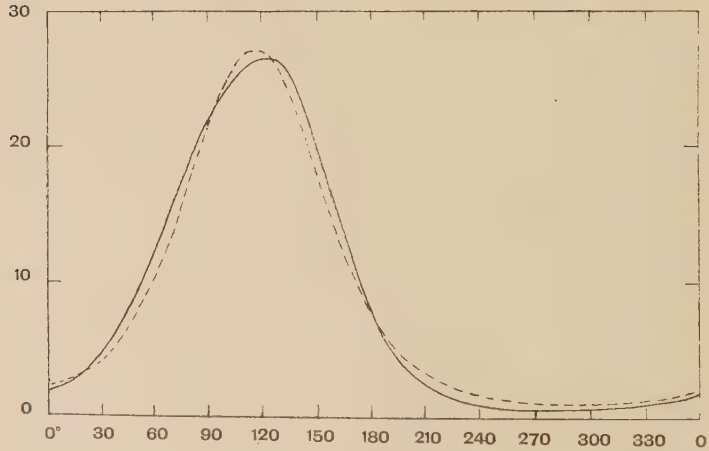

FIG. 2.—Showing the difficulty of separating closely intermingled Drifts.

has not considered is, that when two drift directions are always so close together as the first drift, and the solar drift appear on his diagrams, the resultant curve, although both drifts may be equally strong in number, presents itself with great approximation as a one-drift phenomenon. A fictitious case may illustrate the point. Suppose two drifts, one of which is moving in the direction

$\theta_0 = 90^\circ$ with a velocity $h\mathrm{V} = 1{\cdot}05$, the other in the direction $\theta_0 = 135^\circ$ with a velocity $h\mathrm{V} = 1{\cdot}23$. Each drift is supposed to contain 100 stars; the interval $d\theta = 15^\circ$. With these data, Eddington's formulæ enable us to compute the number of stars contained in each angular spacing $d\theta$. The resultant numbers are shown in the drawn-out curve of fig. 2. Now there is undoubtedly a lack of symmetry in this curve, which shows that we have not to deal with a *pure* one-drift phenomenon. But the deviations are so slight that, not knowing the real nature of the phenomenon, we should certainly conclude that the curve was due to one single star-drift, and would try to represent it by a theoretical one-drift curve, such as the dotted curve of our figure. Our empirical result would thus be a single drift moving in the direction $\theta_0 = 116^\circ$ with a velocity $h\mathrm{V} = 0{\cdot}90$, and containing 200 stars, a result widely different from the actual premises of the case. It was doubtless due to such failures of the analytical method in disentangling closely interwoven drift motions that we arrived at the erroneous conclusion with regard to the distribution of Drift II. stars pointed out by Eddington.

Evidently, then, the analytical method by which a mixture of drifts is separated into its individual components is not always reliable, and therefore an attempt to trace the existence of a third drift, always closely associated with one of the two others, by an analysis of the numbers of stars in each individual area, is bound to prove abortive.

But a more definite answer may be given, if we approach the question in the opposite direction, by making a definite assumption with regard to the direction and velocity of the third drift, and then find how far the representation of the star numbers in the separate areas is improved by its inclusion.

Suppose we have two star drifts and we know the positions of their apices as well as their velocities $h\mathrm{V}$. Then, for any area, the number of stars of Drift I. lying between θ and $\theta + d\theta$, according to Eddington, is expressed by

$$\mathrm{B}d\theta \left\{ \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^{\infty} e^{-x^2} dx \right\} = \mathrm{B}d\theta \,.\, f(\tau),$$

where $\mathrm{B}d\theta$ is a constant depending on the total number of stars of the drift and the width of the angular spacing within which the stars are counted. θ is the inclination of the centre line of any particular spacing to the direction of the drift motion. The latter being known from the assumed position of the apex, θ and also τ are known, since $\tau = h\mathrm{V} \cos \theta$. We may therefore compute $f(\tau)$ for all values of θ, and obtain for each θ an equation of condition

$$\text{Observed number} = a f_1(\tau) + b f_2(\tau),$$

in which the only unknowns are a and b, which are readily determined by a least square solution. We may even reduce the

number of unknowns from two to one if we express the observed numbers in percentages of the total number of stars, and multiply the $f_1(\tau)$ and $f_2(\tau)$ by appropriate factors so that $\Sigma f_1(\tau) = \Sigma f_2(\tau) = 100$.

This makes $a + b = 1$, and leads to the equation of condition

$$[\text{Obs.} - f(\tau_2)] = a[f(\tau_1) - f(\tau_2)].$$

Similarly, in the case of three drifts, the equation of condition assumes the form

$$[\text{Obs.} - f(\tau_3)] = a[f(\tau_1) - f(\tau_3)] + b[f(\tau_2) - f(\tau_3)]$$
$$a + b + c = 1$$

involving now two unknowns, a and b; c, the proportional number of stars of the third drift, being determined by the second relation. Now, as regards the apex of the supposed third drift, it was decided, in view of the results derived by Kapteyn and Eddington with regard to the Orion type stars, to assume the solar antapex as their point of convergence. Further, the theoretical value of hV was found by the following consideration. The most probable value of the average peculiar speed of the spectroscopic stars (excluding the Orion type) was found above to be $\pm 13 \times 0{\cdot}8 = \pm 10{\cdot}4$ km. per second; for the Orion stars by Kapteyn and Frost $\pm 6{\cdot}3$ km.; the ratio $= 1{\cdot}65$. On the other hand, Eddington found the velocity of the motion towards the solar antapex for stars (excluding the Orion type) $h\text{V} = 0{\cdot}9$. Hence, for the Orion stars and those physically associated with them, we should expect $h\text{V} = 0{\cdot}9 \times 1{\cdot}65 = 1{\cdot}5$. The hypothetical constants of the third drift, which in future will be denoted by O, are therefore

$$\text{Drift O: Apex R.A.} = 90^\circ, \quad \text{Decl.} = -36^\circ; \quad h\text{V} = 1{\cdot}5.$$

With these constants the values of $f(t)$ were computed for each area for all values of θ by means of Eddington's table on p. 37 of his paper, "The systematic motions of the Stars" (*M.N.*, vol. lxvii., November 1906).

As regards the apices of the principal two drifts, it is clear that if Drift O really exists, and has passed unrecognised in former investigations, the apices formerly derived cannot be strictly correct. The tendency of the Drift O stars would be to displace the direction of either drift motion towards their own. The mean of all former researches (see Eddington, *M.N.*, vol. lxxi. p. 41) places the apex of

$$\text{Drift I. at R.A.} = 89^\circ, \quad \text{Decl.} = -13^\circ;$$
$$\text{and Drift II. at R.A.} = 272^\circ, \quad \text{Decl.} = -55^\circ.$$

In the present work the positions of these apices and the values of hV were found to be

$$\text{Drift I. R.A.} = 90^\circ, \quad \text{Decl.} = 0^\circ; \quad h\text{V} = 1{\cdot}5.$$
$$\text{Drift II. R.A.} = 270^\circ, \quad \text{Decl.} = -49^\circ; \quad h\text{V} = 0{\cdot}9.$$

We need not enter here upon the method of trial and error by which these apices were found. The points we wish to set forth are—

(*a*) The three-drift hypothesis represents the observed distribution of stars with greater accuracy than the assumption of either one or two drifts.

(*b*) The distribution of the Drift O stars points to a close relationship with the Orion type stars.

(*c*) The inequalities in the distribution of the drift stars as found from the present hypothesis account qualitatively and quantitatively for the systematic character of the residuals in the radial and transverse motions which have been demonstrated above.

It might be urged that the introduction of a third drift, by involving additional unknowns, was bound to lead to a better agreement between observation and theory. But it must not be forgotten that the more rigorous geometrical conditions imposed on the problem lead to a considerable reduction in the number of quantities which have to be evaluated from the observed facts. Thus the representation of the observed distribution in the six standard areas, which will presently be discussed, is based on twenty-one unknowns, viz. two coordinates of each of the three apices, three velocities hV, and two further unknowns a and b for each area. Eddington's analysis based on the two-drift hypothesis involves five unknowns for each area, if the star numbers are expressed in percentages of the total number, *i.e.* thirty unknowns for the six areas.

Of the areas to be examined, those in $\pm 50^\circ$ declination were taken from Eddington's paper. Their centres are situated at 0^h, 4^h, 8^h, 12^h, 16^h, and 20^h R.A. For the equatorial areas, whose centres lie at 1^h, 3^h . . . 23^h and 0° declination, the star counts which had already been made here before the appearance of Eddington's paper were utilised. The Orion stars are included, but stars with proper motions greater than $30''$ per century were omitted, the reason being to make the material as nearly as possible comparable with the material used in the determination of the precessional constant.

For a general test of the theory it is, of course, advisable to reduce as much as possible the purely accidental disfigurements of the observed curves by combining such areas in which the apical directions are either the same or can be made identical by counting the θ's in the opposite sense. Owing to the fact that the R.A. of the three apices are very nearly 6^h or 18^h, we may thus combine

Decl. 0°: 1^h, 11^h, 13^h, 23^h	Decl. $\pm 50^\circ$: 0^h, 12^h
3^h, 9^h, 15^h, 21^h	4^h, 8^h
5^h, 7^h, 17^h, 19^h	16^h, 20^h

The star numbers in these six standard areas were now treated

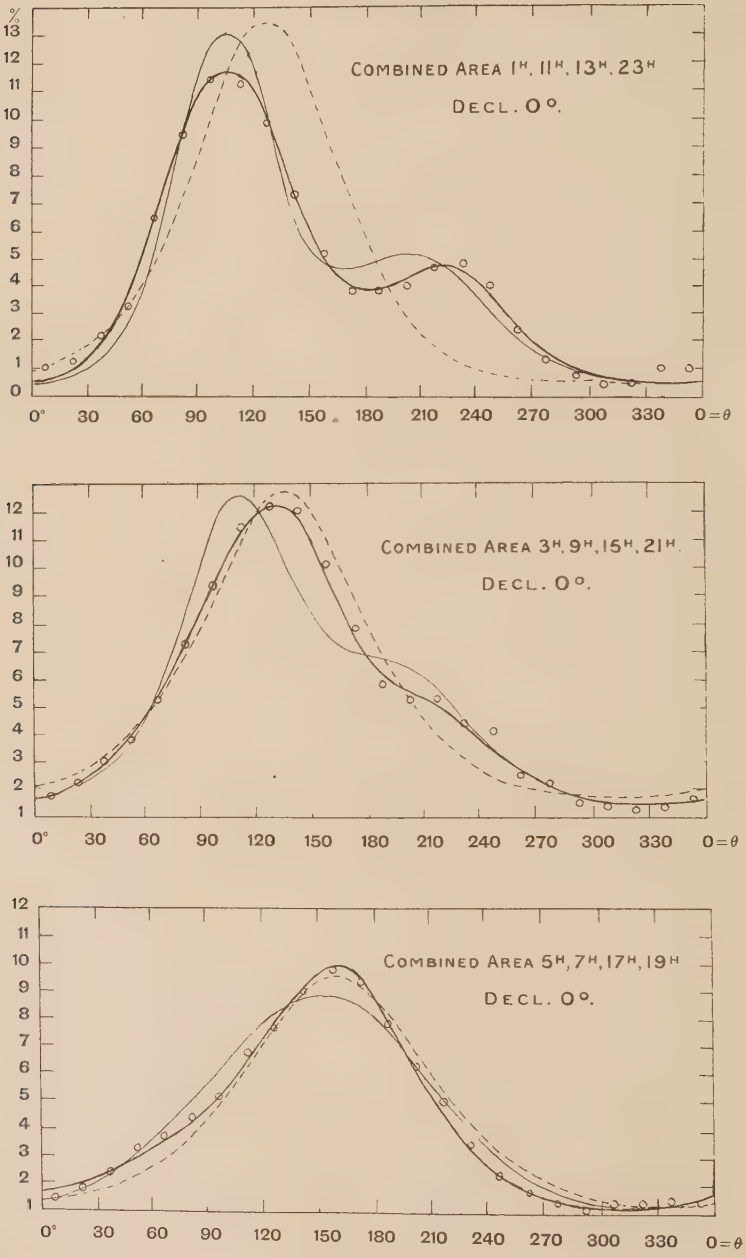
%
COMBINED AREA 1H, 11H, 13H, 23H
DECL. 0°.
COMBINED AREA 3H, 9H, 15H, 21H.
DECL. 0°.
COMBINED AREA 5H, 7H, 17H, 19H
DECL. 0°.
0° 30 60 90 120 150 180 210 240 270 300 330 0=θ

FIG. 3.

in the manner described above, the values of $f(\tau)$ being computed from the assumed positions of the apices and values of hV,* and the unknowns a and b, expressing the proportional numbers of stars of Drifts I. and II., being evaluated by a least square solution.

The outcome of these calculations is condensed in a diagram-

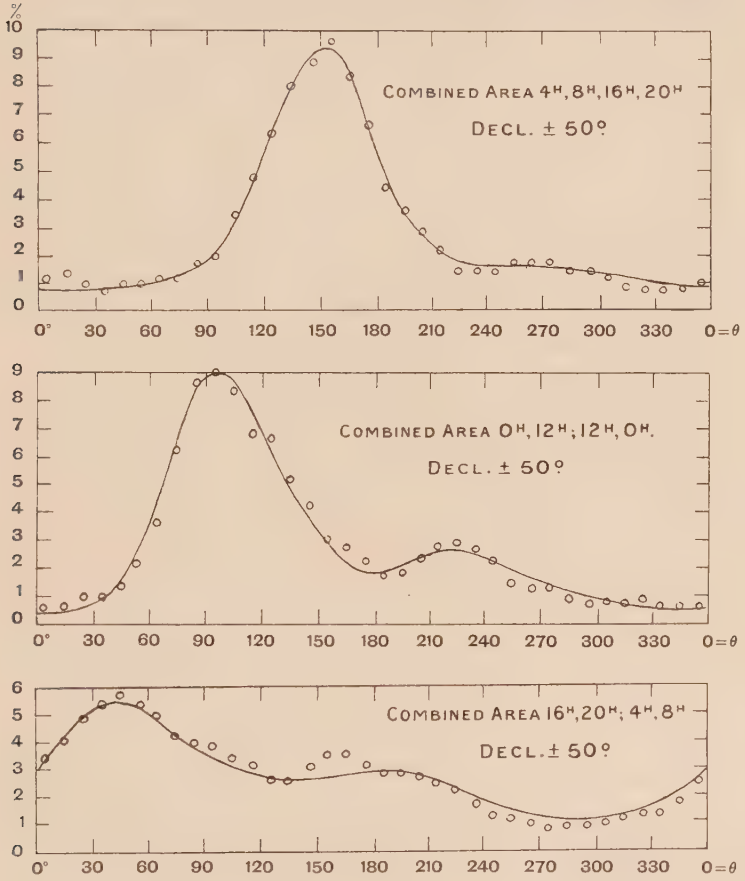

FIG. 3 (*continued*).

matical form in fig. 3. The axis of abscissæ represents the angles θ counted from north over east, and the ordinates show the theoretical percentage number of stars contained within each angular spacing $d\theta$; the latter being 15° wide in the equatorial areas, and 10° in Mr. Eddington's curves. The observed percentage

* It must be mentioned that the value of this quantity employed in the calculations is not the hV given in the text, but hV sin α, where α is the angular distance between the centre of the area and the apex.

numbers are represented by small circles, while the strongly drawn curve shows the numbers resulting from calculation.

It will be admitted that the agreement between theory and observation is, on the whole, as close as can be expected, except a distinctly local disturbance in the last curve at $\theta = 160°$, of which none of the hypotheses can give an account.

Not so satisfactory is the manner in which the facts are represented by the two-drift hypothesis. Calculations based on Eddington's determination of the two apices and of his values for hV were made for the three equatorial areas, and are shown in the thin-line curves of the diagrams.

Lastly, similar calculations were made on the supposition of one drift only, and based on the determination of the solar antapex R.A. $= 90°$, Decl. $- 36°$, and Eddington's value $h\mathrm{V} = 0{\cdot}9$. These calculations are represented by the dotted line curves.

A comparison of these curves is interesting. In the 1^h area the one-drift curve appears to be hopelessly wrong, the two-drift curve a fair approximation, but not nearly so good as the curve calculated from the present theory.

In the 3^h and 5^h areas, however, the one-drift curve, while distinctly inferior to the three-drift curve, is even better than the two-drift curve. This fact is remarkable in so far as it may help to explain Professor Boss's conclusion that the one-drift hypothesis is sufficient to account for the observed facts in the greater part of the sky, although we must confess that we have not been able to ascertain on what grounds his conclusion is based.

The analogy between the present results and those obtained from the radial motions is apparent. From the latter we found that there are two distinct families of stars moving in different directions. They are closely, though not evenly, intermixed, and form a star cloud of unknown dimensions. If the Sun's motion with regard to this cloud as a whole is eliminated, the systematic motions of the individual members of the cloud are not reduced to rest, but one part of the cloud is observed to move with considerable velocity in one direction and the other part in the opposite direction. But it was found that not all the stars could be classified among these two star drifts. A certain class of stars, spectroscopically characterised as the B type or Orion type, showed a notably different behaviour. A precise geometrical idea of their peculiar motions may be gained from the following consideration, the *dynamical* probability of which will, however, not be advocated.

Suppose the density of the two drifts to be the same, also assume that certain stars of the one drift have collided with stars of the other drift, and have become permanently united. According to mechanical principles, the motion of the centre of gravity of the pair is not affected by the collision, and hence the motion of the Sun with reference to the double system after union must be identical with its motion with reference to the two drifts as a whole. The apparent motion of the combined stars is therefore directed towards the same point towards which the stars as a

whole converge, viz. the solar antapex. But the translatory motion of the two stars relatively to their common centre of gravity being destroyed by the collision, the combined stars are obviously reduced to rest by eliminating the solar motion. This is an exact geometrical interpretation of the systematic motions of the Orion type stars. They form a *single* drift, whose apparent motion is directed towards the solar antapex. The fact that the Orion type stars exhibit this peculiar drift in perhaps its purest form does, however, not preclude the presence of stars of the same characteristic motion in other types as well, where they cannot be separated from the more prevalent members of the two other drifts.

We have now traced with tolerable certainty the same phenomena in the transverse motions by demonstrating the better agreement between theory and observed facts if such a drift as is represented by the Orion type stars is taken into consideration. We shall proceed to strengthen the evidence in favour of the existence of this third drift by pointing out a close relationship in distribution between the Drift O and the Orion type stars.

Distribution of the Drifts.—In order to examine how the stars of the three drifts are mixed in different regions of space, we have to revert to the individual areas and to ascertain by the method above described the quantities a, b, c, the proportional number of stars belonging to each drift, and further, by multiplication into the total number of stars, the absolute star numbers of each drift. The result of our calculations is shown in Table V. The absolute star numbers are reduced to the same unit area, which comprises 1700 square degrees.

TABLE V.

Declination +50°.				Equator.				Declination −50°.			
Area. h	Drift I.	II.	O.	Area. h	I.	II.	O.*	Area. h	I.	II.	O.
0	135	75	63	23 } 1 } *	82	88	46	0	52	77	66
4	160	62	9	3	38	62	88	4	59	30	127
8	0	111	129	5	113	19	177	8	10	31	272
12	58	61	78	7	146	6	129	12	56	62	112
16	67	57	78	9	59	31	112	16	42	38	112
20	85	85	151	11 } 13 } *	72	48	47	20	118	59	0
Mean	84	75	85	15	22	38	137	Mean	56	49	115
				17	34	22	159				
				19	94	127	55				
				21	77	78	84				
				Mean	74	55	94				

* In counting the stars, these two areas had been treated jointly before it was decided to subdivide into smaller groups.

The first striking fact is the large percentage of Drift O stars. In the general mean we find that 32 per cent. of the stars belong to Drift I., 25 per cent. to Drift II., and 43 per cent. to Drift O.

Another remarkable feature is that when the Drift O stars increase, it is at the expense of the two other drifts. Notice the gradual increase in the number of the former from north to south, and the corresponding decrease in the latter. The impression at first sight is that the O stars have been formed by a combination of stars of the Drifts I. and II. This feature is specially marked in the southern hemisphere, where we notice a most pronounced inequality in R.A. in the O stars, which is faithfully accompanied by the inverse phenomenon in the other stars.

The areas grouped along the parallel $-50°$ are specially in

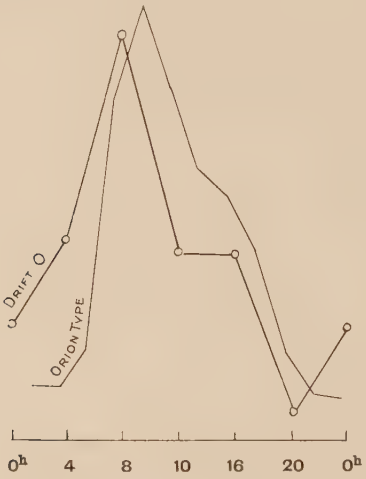

Fig. 4.—Relative Frequency of Drift O and Orion Type Stars along the parallel of $-50°$ Decl.

teresting because they cover regions where the Orion stars are particularly abundant.

The frequency of the Orion stars along that parallel has been examined on the basis of the data supplied by the Harvard Southern Catalogue of brighter spectroscopic stars, and we exhibit the parallelism between the frequency of the Orion stars and stars of the O Drift in fig. 4. A similar correspondence on a lesser scale obtains at the equator, where both Orion and Drift O stars are most frequent in the galactic regions near 6^h and 18^h R.A. Along the parallel $+50°$ the Orion stars show only slight accumulations near 20^h and 4^h R.A. The distribution of Drift O stars is erratic, though there are maxima at 20^h and 8^h R.A.

On reviewing all these facts, the association of the two classes of stars in distribution cannot but point to a relationship. We can hardly avoid the conclusion that the Orion stars are members of a

much greater family, with which they share the particular features of motion which distinguish them from the remainder of the stars.

Of special interest for the subsequent part of our investigation is the drift distribution along the equatorial belt. In fig. 5 the plane of the paper is supposed to represent the equator, the lengths of the radii drawn from S the Sun towards the centres of the areas marking the frequency of drift stars in that particular direction. It will be noticed that Drift I. stars are most frequent in galactic regions. In the case of the O stars the diameter of greatest frequency does not quite coincide with the line passing through the opposite galactic centres. It is perhaps more than coincidence that they are most abundant in those regions where

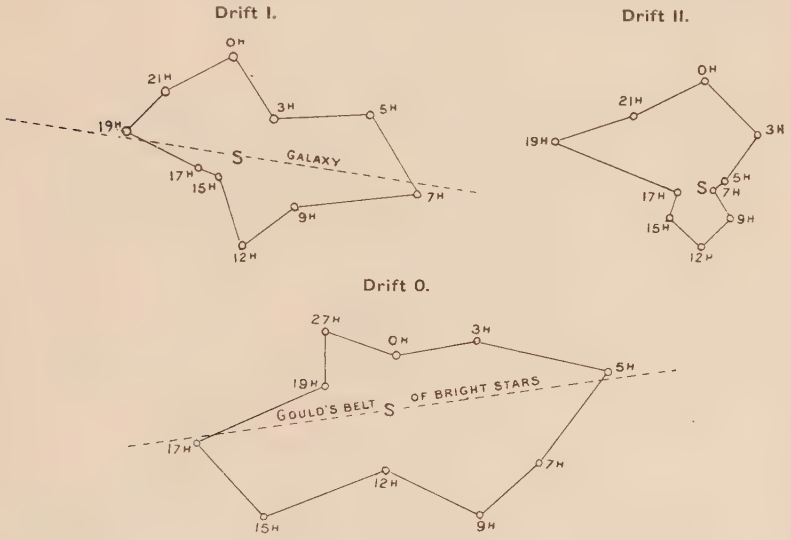

Fig. 5.—Distribution of Drifts along the Equator.

the belt of bright stars known as Gould's belt intersects the equator. The most remarkable feature, however, is presented in the highly unsymmetrical distribution of the stars of the second drift. It is this pronounced inequality to which attention was drawn in the previous paper (*M.N.*, vol. lxx., November 1909), and which there led to the conclusion that the systematic character of the residuals in the radial motions may be due to inequalities in the distribution of the stars of the second drift, a conclusion borne out by the results of the next paragraph. We notice again the "inverse" relation between Drift II. and Drift O, the diameter of *greatest* frequency of the latter coinciding with the diameter of least frequency of the former. This relation, however, is not marked in the first drift, where apparently a preponderating influence of the galaxy exerts itself.

Effect of inequalities in the distribution of the Drift Stars on the radial and transverse motions of the Stars as a whole.

Let a, b, c be the proportional numbers of the drift stars for a given area, subject to the condition

$$a + b + c = 1.$$

Let a_0 be the mean value of a taken over the whole sky and write

$$a = a_0 + \Delta a,$$

where Δa is an unknown function of the position of the area, and similarly

$$b = b_0 + \Delta b$$
$$c = c_0 + \Delta c.$$

The systematic part of the mean radial motion of all the stars contained in the area is then

$$\begin{aligned} V = {} & [(a_0 + \Delta a)V_1\xi_1 + (b_0 + \Delta b)V_2\xi_2 + (c_0 + \Delta c)\xi_3] \cos\alpha \cos\delta \\ & + [(a_0 + \Delta a)V_1\eta_1 + (b_0 + \Delta b)V_2\eta_2 + (c_0 + \Delta c)\eta_3] \sin\alpha \cos\delta \\ & + [(a_0 + \Delta a)V_1\zeta_1 + (b_0 + \Delta b)V_2\zeta_2 + (c_0 + \Delta c)\zeta_3] \sin\delta \end{aligned}$$

where ξ_1, η_1 and ζ_1 are the direction cosines of the apparent motion of Drift I., etc.

Subtracting the mean solar motion, we are left with the residuals

$$\begin{aligned} \Delta V = {} & [\Delta a V_1\xi_1 + \Delta b V_2\xi_2 + \Delta c V_3\xi_3] \cos\alpha \cos\delta \\ & + [\Delta a V_1\eta_1 + \Delta b V_2\eta_2 + \Delta c V_3\eta_3] \sin\alpha \cos\delta \\ & + [\Delta a V_1\zeta_1 + \Delta b V_2\zeta_2 + \Delta c V_3\zeta_3] \sin\delta. \end{aligned}$$

The first term may be neglected, since $\xi_1 = \xi_2 = \xi_3 = 0$, and the last term vanishes if the centre of the area is chosen on the equator. Hence in this case

$$\Delta V = [\Delta a V_1\eta_1 + \Delta b V_2\eta_2 + \Delta c V_3\eta_3] \sin\alpha.$$

Now the preceding investigation supplies the numerical values of the bracket term for a number of such equatorial areas, and thus enables us to find the residuals ΔV which would arise from unequal distribution. If these theoretical values of ΔV agree with those directly obtained from observation, our assertion that the observed anomalies are due to that cause attains a high degree of probability.

Table VI. shows the percentage numbers a, b, c, as found from star counts in the area 1^h, 2^h, 5^h . . . . . 23^h, and between $\pm 30°$ declination, after smoothing out the more accidental fluctuations by the usual process of combining three neighbouring values into one.

TABLE VI.

Area. h	a.	b.	c.	Δa.	Δb.	Δc.
1	0·319	0·381	0·300	+0·001	+0·132	-0·133
3	0·315	0·266	0·419	-0·003	+0·017	-0·014
5	0·362	0·137	0·501	+0·044	-0·112	+0·068
7	0·393	0·079	0·528	+0·075	-0·170	+0·095
9	0·400	0·144	0·456	+0·082	-0·105	+0·023
11	0·355	0·224	0·421	+0·037	-0·025	-0·012
13	0·294	0·236	0·470	-0·024	-0·013	+0·037
15	0·217	0·185	0·598	-0·101	-0·064	+0·165
17	0·200	0·249	0·551	-0·118	0·000	+0·118
19	0·270	0·295	0·435	-0·048	+0·046	+0·002
21	0·345	0·396	0·259	+0·027	+0·147	-0·174
23	0·360	0·381	0·259	+0·042	+0·132	-0·174
Mean	0·318	0·249	0·433			

In each area the conditions $a+b+c=1$ and $\Delta a+\Delta b+\Delta c=0$ must, of course, be strictly fulfilled.

Expressed in the unit adopted in the preceding investigation we assume

$$V_1=1\cdot5, \quad V_2=0\cdot9, \quad V_3=1\cdot5,$$

and also

$$\eta_1=\sin A_1\cos D_1; \quad \eta_2=\sin A_2\cos D_2; \quad \eta_3=\sin A_3\cos D_3;$$

A_1, D_1, ... being the right ascensions and declinations of the apparent apices. Hence we find

$$\eta_1=1\cdot000; \quad \eta_2=-0\cdot656; \quad \eta_3=0\cdot809;$$

and $$V_1\eta_1=+1\cdot50; \quad V_2\eta_2=-0\cdot59; \quad V_3\eta_3=+1\cdot21.$$

Multiplying these latter respectively into Δa, Δb, Δc, the numbers of the 2nd, 3rd, and 4th columns of Table VII. are obtained.

TABLE VII.

1 Area. h	2 $\Delta aV_1\eta_1$.	3 $\Delta bV_2\eta_2$.	4 $\Delta cV_3\eta_3$.	5 Σ.	6 $\Sigma \sin a$	7 $\Sigma' \sin a$.	8 $25\Sigma' \sin a$.	9 Obs. ΔV. km.	10 Obs. - Theory.
1	+0·002	-0·078	-0·161	0·237	-0·061	-0·090	-2·2	+0·4	+2·6
3	-0·004	-0·010	-0·017	-0·031	-0·022	-0·075	-1·9	-3·5	-1·6
5	+0·066	+0·066	+0·082	+0·214	+0·207	+0·142	+3·7	+3·8	+0·1
7	+0·112	+0·100	+0·115	+0·327	+0·316	+0·257	+6·4	+7·5	+1·1
9	+0·123	+0·062	+0·028	+0·213	+0·151	+0·116	+2·9	+2·7	-0·2
11	+0·055	+0·015	-0·015	+0·055	+0·014	+0·011	+0·3	+0·5	+0·2
13	-0·036	+0·008	+0·045	+0·017	0·044	-0·015	-0·4	-0·2	+0·2
15	-0·151	+0·038	+0·200	+0·087	-0·062	-0·009	0·2	-1·8	-1·6
17	-0·177	0·000	+0·143	-0·034	+0·033	+0·098	+2·5	+2·8	+0·3
19	-0·072	-0·027	+0·002	0·097	+0·094	+0·153	+3·8	+5·8	+2·0
21	+0·040	-0·087	-0·211	-0·258	+0·182	+0·217	+5·4	+4·2	-1·2
23	+0·063	-0·078	-0·211	-0·226	+0·058	+0·061	+1·5	+3·5	+2·0

The fifth column contains $\Sigma = \Delta a V_1 \eta_1 + \Delta b V_2 \eta_2 + \Delta c V_3 \eta_3$, and the sixth column shows these sums multiplied into $\sin \alpha$, which, according to the formula, should be equivalent to ΔV expressed on the scale here adopted. It must, however, be borne in mind that terms of the form $(m \cos \alpha + n \sin \alpha)$ would necessarily combine with the mean solar motion which has been eliminated from the observations. An analysis of the figures of column 6 shows that these terms amount to

$$+0\cdot062 \sin \alpha + 0\cdot013 \cos \alpha.$$

The numerical equivalents of this expression have therefore been subtracted, and the corrected values are given in the 7th column headed $\Sigma' \sin \alpha$. Now these numbers should, barring a constant factor, represent the residuals remaining in the observed radial velocities after subtraction of the mean solar motion, if the stars are grouped symmetrically to the equator. The factor required to make the theoretical and observed values comparable in size is 25. The values of $\Sigma' \sin \alpha$ multiplied into this factor are shown in the 8th column, while the 9th column exhibits the observed ΔV already given in Table I.

The agreement between observation and theory is extremely satisfactory, and must be considered a proof of the correctness of the assertion that the observed inequalities in the radial motions are due to irregularities of distribution of the drifts.

In the unit here chosen the Sun's velocity amounts to 1·02, and is directed towards an apex in R.A. = 270°, Decl. = +33°. Expressed in kilometres, it amounts to 20·8 km. for the generality of stars. The theoretical factor should therefore have been $\frac{20\cdot8}{1\cdot0} = 21$ approximately. The close agreement with that which fits best the observations, viz. 25, shows that the agreement between theory and observation is also *quantitatively* established.

The very pronounced existence of a double periodic term in the ΔV is naturally due to the prevalence of a single period in the values Σ. Analysing the latter with regard to $\sin \alpha$ and $\cos \alpha$ we find the single periodic terms

$$0\cdot189 \sin \alpha - 0\cdot179 \cos \alpha.$$

Multiplying into $\sin \alpha$ we obtain

$$0\cdot095 + 0\cdot131 \cos 2(\alpha - 112°),$$

or expressed in kms.,

$$2\cdot4 + 3\cdot3 \cos 2(\alpha - 112°).$$

The two maxima in the residuals of the radial motions therefore appear, according to theory, at $\alpha = 7^h\cdot5$ and $19^h\cdot5$, which is in accordance with observations (see fig. 1).

In the transverse motions the residuals in the component

$\mu_\alpha \cos\delta$ for stars grouped symmetrically to the equator are generally expressed by

$$\Delta(\mu_\alpha \cos\delta) = -\varpi[\Delta a V_1 \xi_1 + \Delta b V_2 \xi_2 + \Delta c V_3 \xi_3] \sin\alpha + \varpi[\Delta a V_1 \eta_1 + \Delta b V_2 \eta_2 + \Delta c V_3 \eta_3] \cos\alpha,$$

hence

$$\frac{1}{\varpi}\Delta(\mu_\alpha \cos\delta) = \Sigma \cos\alpha = -0''.089 + 0''.131 \cos 2(\alpha - 67^\circ).$$

To convert the numerical terms into seconds of arc we have to consider that the velocity of the parallactic solar motion was found by Boss to be $3''.85$ per century. Hence the factor of reduction is $\frac{3.85}{1.02} = 3.8$, and the residuals in $\mu_\alpha \cos\delta$ are theoretically expressed by

$$-0''.35 + 0''.50 \cos 2(\alpha - 67^\circ).$$

This expression applies, strictly speaking, only to stars situated on the equator; for stars in general the factor $\cos\delta$ has to be added.

Comparing with the observed residuals we find that the amplitude of the second order term agrees well with that derived by Newcomb from the Bradley stars, but is somewhat in excess of that found from the stars of Boss's Catalogue.

There is also a noteworthy difference in phase, the maxima occurring at about 6^h and 18^h, instead of $4^h.6$ and $16^h.6$ R.A.

It must be borne in mind, however, that in the transverse motions we had to build our theory on an assumption which was not required in dealing with the radial velocities, viz. that the average distances of the stars are the same in all directions. An assumption of this kind is certainly not *a priori* warranted; in fact there is strong evidence militating against it. An inequality in parallax would naturally lead to expressions similar to those resulting from inequality of distribution, and might therefore materially affect the amplitude and phase of the theoretical second order terms. It is interesting to notice that the assumption that the stars at the solar antapex, *i.e.* where they are apparently moving from us, are on the average farther removed than on the opposite side, would be sufficient to bring the observed and computed phases into harmony. The lack of our knowledge of the average distances naturally renders the evidence from transverse motions less conclusive than that derived from the radial motions.

Summarising the results of the present section, the main conclusion is that the existence of the second order terms is incompatible with the assumption of a random distribution of stars. If their presence were confined to the transverse motion, the assumption of an inequality in the average distances might perhaps explain the observed phenomena. But their strongly marked appearance in the radial motions, which could not be interpreted on this assumption, and their very close agreement with the theoretical values derived from the distribution of the drifts, as found from the

star counts of Professor Boss's General Catalogue, lends strong support to the conclusion that they are the result of unequal mixture of *several* drifts.

The Maxwellian Law, and the relation between mass and average peculiar motion.—A review of the various attempts recently made to unravel the intricacies of star motion on the basis of Eddington's mathematical analysis must convince us that the applicability of the Maxwellian law to the cosmic motions, at least within the separate drifts, has been fully vindicated by the results. A critical examination of the law on theoretical grounds, which is in course of preparation for press, has also led to the conclusion that its application to the kinetic theory of stars cannot be objected to. In fact, it appears to be applicable, as Jeans has demonstrated, to all forms of conservative mass systems, irrespective of their dimensions and the peculiar mode of mutual interaction between the individual members, either by direct collisions or by attractive forces.

One important point, however, has so far been overlooked, viz. the necessary relation between mass and peculiar motion. We are, of course, familiar with the important conclusion drawn from the Maxwellian law, that in a mixture of two gases, say hydrogen and oxygen, the lighter molecules have greater average speeds than the heavier ones. Their relation to each other is determined by the important law known as the "equipartition of energy," which enunciates that the kinetic energy of the system is evenly divided between the heavier and lighter members. If in a system, in which the Maxwellian law obtains, we were able to examine the motions of the heavier and lighter members separately, we should find that the average kinetic energy of the former is equal to that of the latter, and hence that their average speeds are inversely proportional to the square roots of their masses.

The mathematical demonstration is as follows. According to the Maxwellian law, the number of molecules of mass m whose velocities lie between u, v, w and $u + du$, $v + dv$, $w + dw$, is

$$A \cdot e^{-k^2 m(u^2+v^2+w^2)} du \cdot dv \cdot dw,$$

A and k^2 being constants.

The kinetic energy of a single molecule of mass m is $\frac{1}{2}m(u^2+v^2+w^2)$. Hence the total energy of all the molecules

$$E = \tfrac{1}{2}\Delta m \int_{-\infty}^{+\infty}\int\int e^{-k^2 m(u^2+v^2+w^2)}(u^2+v^2+w^2)du \cdot dv \cdot dw$$

$$= \frac{1}{8}\frac{A}{k_2}\sqrt{\frac{\pi}{k^2 m}}\left[\int_{-\infty}^{+\infty}\int e^{-k^2 m(v^2+w^2)} dv \cdot dw + \int_{-\infty}^{+\infty}\int e^{-k^2 m(u^2+w^2)} du \cdot dw \right.$$

$$\left. + \int_{-\infty}^{+\infty}\int e^{-k^2 m(u^2+v^2)} du \cdot dv\right].$$

The integrals in brackets being identical we may write

$$E = \frac{3}{8}\frac{A}{k^2}\sqrt{\frac{\pi}{k^2 m}}\int_{-\infty}^{+\infty}\int e^{-k^2 m(v^2+w^2)}dv\,.\,dw.$$

On the other hand, the total number of molecules is

$$N = A\int_{-\infty}^{+\infty}\int\int e^{-k^2(u^2+v^2+w^2)}du\,.\,dv\,.\,dw = \tfrac{1}{2}A\sqrt{\frac{\pi}{k^2 m}}\int_{-\infty}^{+\infty}\int e^{-k^2 m(v^2+w^2)}du\,.\,dv.$$

Therefore the average kinetic energy

$$\bar{E} = \frac{E}{N} = \frac{3}{4k^2}.$$

In words: the average kinetic energy is independent of the mass. If Ω denotes the average speed we thus find

$$m\Omega^2 = \text{const.},$$

i.e. *the average speed is inversely proportional to the square root of the mass.*

Now the question arises: Are we in a position to sort out, so to speak, the stars according to their masses in order to test the validity of this important law of equipartition of energy? It is maintained that this sorting process is, in some degree at least, accomplished by the spectroscope; *i.e.* that the spectral type of the star offers an indication of its average mass. On *a priori* grounds it is certainly not illogical to associate the rate of development from earlier to more advanced types with the mass of the star. If stars of different masses have started on their career as luminous bodies at the same time, we should, *a priori*, expect the lighter stars to cool down more quickly, and hence to arrive at the more advanced spectroscopic stages sooner than the heavy stars. From this reasoning, then, the conclusion would be reached that the average masses of the early spectral types should be greater than those of the more advanced types. We are now in a position to test this conclusion by means of the principle of equipartition. If our reasoning is correct, the stars of the early types must possess smaller average speeds than those of more advanced types.

Now this is in exact accordance with the results of Kapteyn's investigation, shown in Table VIII.

TABLE VIII.

Type.	Average peculiar speed. km.	No. of Stars.
B to B_9	6·0	64
A to A_5	11·2	18
F to F_8	14·5	17
G to G_5	12·6	26
K to K_5	15·4	55
M*a*	19·3	6

Of course, too much importance must not be attached to an *a priori* reasoning, however convincing it may sound; a satisfactory proof of the assertion can only be supplied by the evidence of facts.

Unfortunately the material from which a knowledge of the masses may be derived is very meagre, but what there is appears to support the present view most emphatically.

According to Campbell (Second Catalogue of Binary Stars, *Lick Obs. Bull.*, No. 181), there are seven spectroscopic binaries of the Orion type and nine of types A–G of which the combined mass of the system $(m_1+m_2)\sin^3 i$ could be determined. These are stars in the spectra of which the displacements of lines of *both* components could be observed (Group I.).

The material may, however, be considerably increased by including the binaries in which only one component was spectroscopically visible, and which therefore determine the relation of the two masses $\frac{{m_1}^3}{(m_1+m_2)^2}\sin^3 i$ (Group II.).

It is a well-known fact that the ratio between the masses of the two components is very nearly the same for all classes of stars. Thus we find for

Spectroscopic binaries	Orion type	$m_1/m_2 = 0{\cdot}77$
	Other types	,, 0·83
Visual binaries (Boss)		,, 0·81

Hence we may assume that the ratios of the average values of $\frac{{m_1}^3}{(m_1+m_2)^2}\sin^3 i$ for different types are practically identical with the ratio of their combined masses, the quantity which we desire to find.

From Group I. we derive

Orion type $(m_1+m_2)\sin^3 i = 16{\cdot}3 \pm 3{\cdot}4$.

Types A–G . . . $1{\cdot}9 \pm 0{\cdot}3$.

Ratio of average mass of Orion type to that of other types = 8·6.

Square root of ratio = 2·9.

Group II. has been subdivided into three groups according to the periods of revolution of the binaries. It was found—

for periods $< 5^d$	—Ratio of average mass of Orion type to that of other types = 8·8	No. of stars = 29
,, ,,	between 5^d and 30^d . = 4·6	,, 24
,, ,,	$> 30^d$ . . . . = 6·2	,, 20
	Mean ratio . . . . = 6·5	
	Square root of mean ratio . = 2·5	

So far as the evidence goes, there seems to be good reason to

assume that the Orion type stars are on the average more massive than the stars of more advanced type.

Now, according to the Maxwellian law, the ratio of the peculiar speeds of these types should equal the inverse square root of the ratio of their masses, *i.e.*—

Average speed of Orion star : average speed of later types

$$= 1 : 2{\cdot}5 = 0{\cdot}40.$$

From the preceding Table VIII. we find

$$6{\cdot}0 : 14{\cdot}3 = 0{\cdot}42.$$

The relation between spectral type and mass is also borne out by the results obtained by Mr. Russell (*Astron. Journ.*, vol. xxvi., Nos. 18–19) from a discussion of visual binaries. From six stars of spectral type A–G he finds the average mass 2·4 in good agreement with the value obtained from spectroscopic binaries of the same type. But three stars of type $K_5 - Ma$ yield a considerably smaller value, viz. 0·3. The evidence, considered by itself, would perhaps appear of little value owing to the fewness of stars, but, taken in conjunction with the preceding data, assumes some significance.

Another important fact pointing in the same direction, and fully borne out by the researches of Kapteyn and Russell, is the relation between type and intrinsic brightness. Knowing the apparent magnitude m and parallax π, the absolute magnitude M—*i.e.* magnitude at distance for which $\pi = 0''{\cdot}1$—is found from the well-known equation

$$M = m + 5(1 + \log \pi).$$

The following table shows the result of Kapteyn's investigation of the brighter stars of various types for which he determined the average parallaxes by means of his ingenious statistical method :—

	Observed App. Mag.	Observed Aver. Parallax.	Computed Absolute Mag.
		″	
Orion type	5·0	0·0066	−0·9
Type A	5·0	0·0098	0·0
Type F–K	5·0	0·0224	+1·8

Mr. Russell's result refers to fainter stars of large proper motions for which direct determinations of parallax have been made. He finds—

Type.	Observed App. Mag.	Observed Parallax.	Computed Absolute Mag.
		″	
F_8	7·0	0·044	5·2
G, G_2	7·8	0·029	5·1
G_5	8·6	0·064	7·6
K	7·4	0·119	7·8
K_5	8·2	0·254	10·2
M	8·3	0·221	10·0

The result is in both cases undoubtedly that stars of early types are intrinsically brighter than stars of advanced types. And combining this result with the preceding conclusion that the early type stars contain the heavier masses, we may also say that *intrinsic brightness and mass are in direct relationship.*

Are there any facts which bear out this remarkable relation?

If we examine the relation between apparent and absolute magnitude of the generality of stars, irrespective of type, by means of their average parallaxes, we arrive at the data collected in the following table, which are based on Kapteyn's investigations.

Observed App. Mag.	Aver. Parallax.	Computed Absolute Mag.
2·7	0″·0383	0·6
4·1	0·0205	0·7
5·1	0·0147	0·9
6·0	0·0129	1·5
6·9	0·0089	1·7
8·6	0·0063	2·6
10·5	0·0048	3·9

It appears, then, that the apparently fainter stars are also on the average intrinsically fainter, *i.e.* their lustre is reduced, not only on account of greater distance, but also on account of inherent feebler luminosity.

Now, if our previous conclusions are correct, the fainter stars must, on the average, be less massive than the bright stars; and hence, according to the Maxwellian relation between mass and peculiar motion, *the fainter stars must show greater peculiar speeds than the bright stars.*

This is exactly what Campbell found to be the case in his discussion of the radial velocities of the northern stars in *Ap. J.*, vol. xiii., No. 1.

On the whole, then, there is a tolerably strong chain of empirical evidence supporting the view that the principle of equipartition of energy is applicable to the system of bodies constituting the visible universe.

The importance of this principle, from the evolutionary point of view, cannot be too much emphasised. It enables us to weigh the stars—not individually, of course, but on the average—through the medium of their *motus peculiares.* We are in a position to separate the stars in order of weight, and to examine the influence of mass on the character of the spectrum and on the surface luminosity, and there can be little doubt that fresh information thus gained will ultimately further our progress towards the ultimate goal of astrophysical research, viz. to comprehend the cause and true meaning of the evolution of the universe.

Throughout the paper I have endeavoured to let the facts

speak for themselves and to abstain from theoretical speculations. But even the facts can be accepted only at their face value. Though they may be trustworthy in themselves, the mere fact that they are necessarily incomplete may bias the general conclusions derived from them. All we can do is to proceed step by step and as carefully as possible, but to be always prepared that our conclusions may have to be revised on the evidence of additional empirical data.

On the Progress of the New Tables of the Moon's Motion.

By Ernest W. Brown, F.R.S.

1. In the *Monthly Notices* for 1909 December a general plan for the formation of the new tables was outlined. The forms to be given to the single- and double-entry tables were described and the numerical details of the tables for the principal solar terms in longitude were shown. At that time the single-entry tables of the longitude were in process of formation. In the interval, the single- and double-entry tables for the solar terms in both longitude and parallax have been practically finished, and the numerical transformation of the latitude into the form adopted for tabulation has also been completed.* The plans for the tabulation of all three coordinates, previously somewhat indefinite, have been worked out in detail: in some cases changes have been made, but the material differences are not great. The stage which has been reached will be described below.

Four more tables have been added to the 22 single-entry tables in longitude, previously described, and two of the tables have been combined into one, resulting in a net increase to 25 single-entry tables. Tables for dealing with the numerous remainder terms with periods of $\frac{1}{3}$ of a month or less have been devised; as the forms of these are somewhat different from those previously used, they are described in detail. In the previous paper it was suggested that the sum of these terms might be tabulated at half-daily intervals for 100 years; this labour is avoided and the period suggested is increased to 200 years. Although a mechanical device by which the former plan could alone be economically carried out is no longer necessary, it is still a desideratum, though not a necessity, for obtaining the sums at 10-day intervals, and is under consideration.

The tabulation of the parallax is altered to the extent of adding two more single-entry tables.

The most troublesome problem was the tabulation of the latitude. It will be seen that in the form finally adopted the

* See a paper entitled "The Transformation of the Moon's Latitude" in the present number of the *Monthly Notices*.

labour for the ephemeris computer would have been less than that required with Hansen's tables if the same degree of accuracy with respect to the number of terms included had been maintained. Many more terms are, however, included, and their presence may result in a slight increase in the time necessary to obtain the ephemeris for this coordinate. In the interpolations, the use of logarithms has been rendered unnecessary; the great majority of them will demand the addition of less than 20 units to the tabular values.

Tabulation of the Longitude.

2. *Table for the elliptic terms.*—The previous plan called for a single-entry table to include the terms with arguments l, $3l$, $5l$; another for those with arguments $2l$, $6l$; while the term with argument $4l$ was to be included in the double-entry table with second argument $l - D$. It was found, however, that this last term would have made the differences larger than was desirable. An additional single-entry table for the term would have required nearly as much space for the six multiples (that is, three reversible tables) as one non-reversible table which should include them all. Besides the saving of labour for the ephemeris computer in having to extract from one table instead of three, it was found that certain planetary terms could also be included which otherwise would have demanded separate provision. A single non-reversible table, with the given division of the principal argument, would occupy 30 pages.

If negative as well as positive numbers be permitted in this table (that is, if no constant be added to make all the numbers positive), it can be made reversible without loss of accuracy, and a saving of about 15 pages can be effected. The computer will then be expected to add a whole number, say 300,000, to every number he takes from the table. If he had to interpolate, this would be troublesome; but it was provided that he was only to copy the numbers following one another in column when using this table, since the interpolation was made at a later stage. His work is therefore little more than copying the positive numbers and writing down the complements of the negative numbers as they occur.

3. *Additional single-entry tables.*—The new tables are the last four of those given in No. 4 below. They were excluded from the double-entry tables because the size of their coefficients combined with their short periods caused troublesome second and third differences in the interpolations from the values at $D = 0^d$, $0^d.5$, . . . , to the values of D at the required dates.

4. The following table shows the main facts concerning the 25 single-entry tables in their final form.

Single-entry Solar Terms in Longitude.

Arg.	Max. Coef.	Approx. Period.	No. of Periods.	No. of Entries.	Max. Diff.	Max. Ann. Error.
	"	d			"	"
il	22640"	27·55	55×6	9093	8·40	0·4
$(1, 3)(2D-l)$	4586	31·81	335	10657	1·35	·02
$4D-2l$	31	15·91	117	1861	·06	·001
$2D$	2370	14·77	49×6	4341	1·80	·05
$iD(i\neq 2)$	125	29·53	49×2	5788	·17	·01
$2D-l-l'$	206	34·85	258	8990	·08	·000
$l-l'$	148	29·80	71×2	4232	·11	·003
$l+l'$	110	25·62	189	4842	·08	·003
$2F-l$	40	26·88	152	4085	·03	·000
$2D-l+l'$	28	29·26	207	6057	·02	·000
$2F$	412	13·61	311	4231	·31	·003
$2D-l'$	165	15·39	599	9217	·06	·000
$2D+l'$	24	14·19	167	2370	·04	·002
$2D+l$	192	9·61	277	2663	·23	·004
$2F+l$	45	9·11	189	1721	·09	·006
$4D-l$	38	10·08	396	3993	·04	·001
$2D+l-l'$	15	9·87	178	1757	·03	·001
$2D+2l$	14	7·13	299	2131	·03	·001
$(1, 3)\,l'$	668	365·26	25	9131	·25	·004
$2l-2D$	212	205·89	14	2882	·24	·006
$2F-2D$	55	173·31	21	3639	·05	·002
$2F+2D$	6	7·08	179	1267	·02	·001
$2F+2D-l$	9	9·53	133	1267	·03	·004
$2F+2l$	4	6·85	68	465	·03	·002
$4D+l$	2	5·82	31	180	·04	·003

The first three columns give the argument, the approximate coefficient (or greatest coefficient), and the approximate period, respectively. The fourth column shows the number of consecutive periods tabulated; when the convergents are unsuitable the subdivisions (see No. 42 of the previous paper) are shown by the factor. The actual number of entries in each table is given in the fifth column. The sixth column shows the maximum difference for interpolation, so that the amount to be added to the tabular value never exceeds half this amount. The last column shows the maximum error of the function which will have accumulated if 730 successive half-daily values be used without change of the argument; this error is due to the use of a period given by the convergent instead of the actual period and is provided for (see No. 41 of the previous paper). With the adopted plan for the first table the only one not reversible is that for $iD(i\neq 2)$.

5. *Double-entry tables in Longitude.*—No changes in the form or number of these have been made. The five terms now in single-entry tables have been extracted. In most cases, the convergents have been changed, partly owing to this cause and partly to obtain a more effective tabulation. The principal facts concerning the 22 tables in their final form are shown below.

Double-entry Solar Terms in Longitude.

Second Argument.	Max. Coef.	No. of Terms.	No. of Parts.	Monthly Addition.	No. of Entries.
$l-D$	18″·6	39	251*	18	15813
l'	18·0	24	141*	11·4	8883
$l+l'-D$	4·4	12	156†*	23·8	4914
$l-l'-D$	1·1	14	116†*	1·06	3654
$2l+l'-2D$	8·6	12	124	27·81	3906
$2l-l'-2D$	9·7	10	128	8·01	4032
$l+2l'-D$	7·4	8	132	30·81	4158
$l-2l'-D$	2·6	6	100	9	3150
$3l+l'-3D$	0·6	8	50	14·8	1575
$3l-l'-3D$	0·7	9	42	5·64	1323
$2F-2D$	0·6	9	51*	8·69	3213
$2F+l-3D$	1·0	8	38	9·2	1197
$2F-l-D$	6·4	8	76	7·5	2394
$2F+2l-4D$	0·6	7	94	29·5	2961
$2F-2l$	1·3	7	56	1·51	1764
$2F+l'-2D$	2·2	8	80	20·1	2520
$2F-l'-2D$	1·4	6	44	3·94	1386
$2F+l+l'-3D$	0·3	4	24	7·75	756
$2F-l-l'-D$	0·4	5	28	0·5	882
$2F+l-l'-3D$	0·3	5	32	5·16	1008
$2F-l+l'-D$	0·4	5	44	7·9	1386
$2F+3l-5D$	0·3	4	36	13·88	1134

The first three columns are sufficiently explained by their headings. The fourth column shows the number of parts into which the second argument has been divided; a star indicates that the table is not reversible, and a dagger that alternate parts of the second argument are alone tabulated in order that the argument need not be changed for the corresponding double-entry table in parallax. The fifth column gives the exact monthly addition to be made to the second argument when D increases by a period. The last column shows the number of actual entries present in each table. All terms greater than 0″·001 have been included.

6. *Remainder terms.*—The plans previously described for the

majority of the larger planetary terms have not been changed, but new methods for dealing with those terms, both solar and planetary, which have periods of $\frac{1}{3}$ of a month or less, and for which special tables seem inadvisable, have been worked out. It was desired to avoid the necessity of computing and tabulating at half-daily intervals for the period suggested, 100 years.

The new plan involves the formation of four arguments, two at intervals of ten days and two at intervals of fifteen days for 200 years, and a special double-entry table. These tables are described below.

7. *Remainder terms with periods approximating to one-third of a month.*—I take the average period of these terms to be exactly 10 days, so that the motion of the average period is 18° per half-day. The terms are then expressed in the form

$$\Sigma B \sin (18^\circ t + At + A').$$

Here t is the number of half-days from the epoch, $18^\circ + A$ is the true motion per half-day, and A' is the argument at epoch; then A is always small. This expression may be put into the forms

$$2C \sin 18^\circ t + 2S \cos 18^\circ t = K \cos (18^\circ + P) + K \cos (18^\circ + Q),$$

where

$$2C = \Sigma B \cos (At + A'),\ 2S = \Sigma B \sin (At + A'),$$

and K is a fixed constant not less than the maximum value of $2(C^2 + S^2)^{\frac{1}{2}}$.

It is evident that C, S and therefore P, Q will be slowly varying quantities. An examination of the terms to be included (about 40 in number) shows that a ten-day interval will be small enough to avoid the use of second differences. Tables of the arguments P, Q are therefore to be formed at ten-day intervals for as long a period as seems desirable: such tables, extending from 1800–2000 A.D., would demand about 7300 values of P and 7300 values of Q.

The ephemeris computer requires the function for every half-day. Suppose he takes out two consecutive arguments P_0, P_1, ten days apart. He then needs the 21 values

$$K \cos \{i.18^\circ + P_0 + \frac{i}{20}(P_1 - P_0)\} + K, \quad i = 0, 1, \ldots 20.$$

First suppose that $P_0 < 18^\circ$, $P_1 > P_0$. The degree of accuracy is such that P need only be known within 3° of its actual value, while $P_1 - P_0$ will be always less than about 24°. We therefore require a table for this function containing 6 multiples of P, 9 multiples of $P_1 - P_0$ and 20 values of i; it will therefore contain 1080 entries, which will occupy less than one page. All the computer has to do is to find the consecutive arguments P_0, P_1 at the dates he needs, enter this table with these arguments, and he will find the values for 21 consecutive half-days in a column, no interpolation being necessary.

If $P_0 > 18^\circ$ it is found expressed in the form $n.18^\circ + P'_0$; the

computer, instead of starting at the top of a column, starts at the value $i=n$; when the foot of the column is reached he continues from the top of the same or another column until the necessary 21 numbers have been obtained.

If $P_1 < P_0$, a suitable change in the instructions to the computer will allow him to make use of the same table. It may, however, be more simple to double the size of the table so as to include negative as well as positive values of $P_1 - P_0$. This can be done without exceeding one page.

A precisely similar plan is followed with the argument Q. The same table serves for the extraction of the half-daily values.

The arguments will not be expressed in degrees but in terms of that unit which makes the use of the table most simple. The circumference may be divided into 200 parts and the value of P may be given to the nearest even number. The units in this value will then be P'_0 while the tens will be n. The number of entries in the table will be 100 times the maximum difference between two consecutive values in the table giving the arguments P or Q.

8. *Remainder terms with periods approximating to one-quarter of a month.*—The same process is followed. The average period is about $7\frac{1}{2}$ days, so that the average half-daily motion may be taken to be exactly 24°. It will probably not be necessary to tabulate the values of P, Q for these terms at more frequent intervals than once in fifteen days, so that the table giving the half-daily values would contain 30 consecutive values of i. It will thus be larger than the corresponding table in No. 7 by one-half, but the tables for the arguments P, Q will be each less by one-third. About a dozen terms will be included.

9. *Special double-entry table.*—There are some terms of still shorter period which have not been included. The most important of these contain the arguments l, 2F only. The terms with these arguments are

$$-0''{\cdot}055 \sin(l-\phi) - 0''{\cdot}007 \sin(2l-\phi) + 0''{\cdot}080 \sin(3l+2\phi)$$
$$+0''{\cdot}090 \sin(5l+2\phi) - 0''{\cdot}025 \sin(6l+\phi) + 0''{\cdot}011 \sin(6l+2\phi),$$

where $\phi = 2F - 2l$.

The first three of these might have been included amongst the remainder terms previously accounted for, but the third of them is much larger than any other term in the tables of No. 7, and it is therefore more suitably placed here. Since the argument D is not present, such a table cannot be treated with the other double-entry tables, the first argument of which is always D.

By considering the convergents, it is found that 159 periods of l and 4 periods of ϕ are very nearly equal to 4381 days. If we form a table containing the values of the function for 8762 consecutive half-days, and so arrange it that the divisions of the argument ϕ are in consecutive columns, it will be found that these divisions are sufficiently small to avoid the necessity of interpolating between successive columns. The ephemeris computer has there-

fore only to choose the proper column shown by his second argument and the line in that column nearest to his first argument, and interpolate between successive numbers in column with a factor which remains constant for several months. The continuity of the various columns for consecutive half-days is arranged for in the same way as with the single-entry tables. Although this table will occupy only three or four pages the plan seems to admit of improvements.

10. *Neglected terms.*—The only terms in longitude greater than 0″·003 which have not yet been placed in tables are

$$-0''\cdot019\sin(l-4F-2D)-0''\cdot009\sin(2l-l'+2F+2D)$$
$$+0''\cdot007\sin(4l-l'+2D)+0''\cdot006\sin(2F+3l+l')-0''\cdot006\sin(2F+3l-l'),$$

and six terms with coefficients less than 0″·006. The last two of those written and a few others with smaller coefficients were not computed in my theory. They are extracted from Delaunay's results. The omission was intentional, since Delaunay's values for them were sufficiently accurate, and since the computation of them by my method would have involved much labour. The two terms with coefficients 0″·006 can be accounted for by a minute addition to the second argument of the double-entry table, argument $2F+3l-5D$, this addition being solely dependent on the day of the year.

The maximum sum of the remaining terms is less than 0″·06 which may be reduced to 0″·04 if we place the first term in a small single-entry table. The final degree of accuracy being 0″·1, the error is immaterial.

Tabulation of the Parallax.

11. This is quite similar to that of the longitude. Since all terms are cosines, every table is reversible. The 7 single-entry and 14 double-entry tables are exhibited below, the signification of the headings being the same as in Nos. 4, 5. The double dagger indicates that every fourth part only of the second argument is tabulated.

No disposition has yet been made of the remainder terms mentioned in No. 56 of my previous paper.

12. *Single-entry Solar Terms in Parallax.*

Argument.	Max. Coef.	Approx. Period.	No. of Periods.	No. of Entries.	Max. Diff.	Max. Ann. Error.
	″	d			″	″
l	186·5	27·55	55×4	6062	0·096	0·003
D	28·2	29·53	49×2	2894	·056	·004
$2D-l$	34·3	31·81	109	3467	·032	·001
$2D+l$	3·1	9·61	277	2663	·004	·001
$2D-l'$	1·9	15·39	71	1092	·006	·000
$4D-l$	0·6	10·08	65	655	·003	·000
$2D+2l$	0·3	7·13	59	420	·003	·000

13. *Double-entry Solar Terms in Parallax.*

Second Argument.	Max. Coef.	No. of Terms.	No. of Parts.	Monthly Addition.	No. of Entries.
$l-D$	$0''.30$	29	251†	18	3969
l'	·40	17	141†	11·4	2205
$l+l'-D$	1·44	12	156	23·8	4914
$l-l'-D$	1·15	11	116	1·06	3654
$2l+l'-2D$	·10	10	124††	27·81	1008
$2l-l'-2D$	·13	10	128†	8·01	2016
$l+2l'-D$	·05	4	132††	30·81	1071
$l-2l'-D$	·02	5	100††	9	819
$3l+l'-3D$	·01	5	50†	14·8	819
$3l-l'-3D$	·01	4	42†	5·64	693
$2F-2D$	·11	6	51	8·69	1638
$2F+l-3D$	·08	8	38	9·2	1197
$2F-l-D$	·71	7	76	7·5	2394
$2F-2l$	·01	4	56†	1·51	882

Tabulation of the Latitude.

14. *Final form of the Latitude.*—In No. 9 of my previous paper it was suggested that the form which seemed to promise the best results was

$$\text{Lat.} = (k+\text{C})\sin(\text{F}+\lambda+\text{S})+\text{N},$$

where λ consists of certain large terms present in the longitude whose sum will have been obtained in computing the longitude; N consists of terms to be put into single-entry tables and of small terms involving the multiples 3, 5 of F; S, C consist of small terms to be placed in double-entry tables. Further consideration has shown that the formula admitted of improvement.

The expression finally chosen is the following:—

$$\text{Lat.} = (k+\text{C})\{\sin(\text{F}+\lambda+\text{S})+\mu\sin 3(\text{F}+\lambda+\text{S}) + \nu\sin 5(\text{F}+\lambda+\text{S})+\text{N}\}$$

where the forms of k, λ, S, C are the same as before; μ, ν are constants, and N consists solely of single-entry terms.

For tabulation this is expressed in the form

$$\left\{1+\frac{\text{C}+\gamma-(k'-k+\gamma)}{k'}\right\}\{k'\sin(\text{F}+\lambda+\text{S})+k'\mu\sin 3(\text{F}+\lambda+\text{S}) + k'\nu\sin 5(\text{F}+\lambda+\text{S})+k'\text{N}\},$$

where γ is the sum of the constants added to the double-entry table for C and k' is a constant to be chosen at will. The terms in C will be divided by k' before tabulation.

Suppose that the computer has obtained the values of

$(C+\gamma)k'$, $F+\lambda+S$, $k'N$ for any date or set of dates. With the argument $F+\lambda+S$, he enters a table to be given for

$$k' \sin(F+\lambda+S) + k'\mu \sin 3(F+\lambda+S) + k'\nu \sin 5(F+\lambda+S).$$

This table is to be sufficiently extended so that the differences used have at most 3 significant figures, permitting the use of Crelle for the interpolations, and the table is reversible. The sums $k'N$ are then added. The final step is the multiplication by the variable factor

$$\frac{C+\gamma-(k'-k+\gamma)}{k'}.$$

As k' is at our disposal, we may so choose it that $k'-k+\gamma=0$. But it is desirable to keep the factor as small as possible, and to do so it will be better to admit negative as well as positive values of the factor. Hence we should give to k' a value not very different from k, the latter being equal to $18519''{\cdot}830$. The constant $(k'-k+\gamma)/k'$ must then be subtracted after the value of $(C+\gamma)/k'$ has been found.

The value of k' chosen is $10^7 \div 54 = 18518''{\cdot}519$; this was adopted in order to facilitate the construction of the table with argument $F+\lambda+S$, since it did not appear that any other use could be made of the arbitrariness of k'. It may be noted that any change in the observed constant of latitude may be included in the constant $(k'-k+\gamma)/k'$, so that such a change causes no alteration of any of the tables.

The present arrangement of the terms included in λ, the 10 single-entry tables for $k'N$, the 16 double-entry tables for S, and the 10 double-entry tables for C are shown in the following numbers. Owing to the factor k', which is about $1/11$ in circular measure, the value of S expressed in seconds requires one place of decimals less than that of C.

15. *Terms included in* λ.—The terms are those placed in single-entry tables in longitude with arguments il, $i(2D-l)$, iD, $l+2D$, $l\pm4D$, $2l\pm2D$, $2D-l\pm l'$, $2D+l-l'$, $l\pm l'$, $l'\pm2D$.

16. *Single-entry Solar Terms in Latitude* ($k'N$).

Argument.	Coef.	Approx. Period.	No. of Periods.	No. of Entries.	Max. Diff.	Max. Ann. Error.
	″	d			″	″
$F-2D$	526	32·28	130	4196	0·40	0·008
$F-4D$	3	10·13	80	810	·01	·000
$F+l-2D$	44	188·20	5	941	·15	·004
$F+l-4D$	6	16·02	112	1794	·01	·000
$F-l$	21	2190·34	1	2190	·03	·002
$F-l-2D$	31	14·87	171	2542	·04	·002
$F-2l$	25	27·91	53	1479	·11	·004
$F-2l-2D$	2	9·66	205	1979	·01	·000
$F+l'-2D$	23	35·41	39	1381	·05	·000
$F-l'-2D$	11	29·66	47	1394	·03	·001

17. *Double-entry Solar Terms in Latitude* (S).

Second Argument.	Max. Coef.	No. of Terms.	No. of Parts.	Monthly Addition.	No. of Entries.
$l-\mathrm{D}$	$59''$	48	251†*	18	7938
l'	127	19	141*	11·4	8883
$l+l'-\mathrm{D}$	24	13	156†*	23·8	4914
$l-l'-\mathrm{D}$	17	15	116†*	1·06	3654
$2l+l'-2\mathrm{D}$	11	8	124†	27·81	1953
$2l-l'-2\mathrm{D}$	12	9	128†	8·01	2016
$3l+l'-3\mathrm{D}$	1	5	50†	14·8	819
$3l-l'-3\mathrm{D}$	1	3	42†	5·64	693
$l+2l'-\mathrm{D}$	6	5	132††	30·81	1071
$l-2l'-\mathrm{D}$	2	5	100††	9	819
$2\mathrm{F}-2\mathrm{D}$	52	7	51	8·69	1638
$2\mathrm{F}+l-3\mathrm{D}$	10	5	38	9·2	1197
$2\mathrm{F}-l-\mathrm{D}$	85	6	76	7·5	2394
$2\mathrm{F}-2l$	1	3	56††	1·51	441
$2\mathrm{F}+l'-2\mathrm{D}$	2	3	80††	20·1	630
$2\mathrm{F}-l'-2\mathrm{D}$	1	5	44†	3·94	693

18. *Double-entry Solar Terms in Latitude* (C).

Second Argument.	Max. Coef.	No. of Terms.	No. of Parts.	Monthly Addition.	No. of Entries.
$-\mathrm{D}$	$5''·7$	42	251†	18	3906
l'	1·3	16	141†	11·4	2205
$l+l'-\mathrm{D}$	2·1	10	156††	23·8	1260
$l-l'-\mathrm{D}$	1·5	13	116†	1·06	1827
$2l+l'-2\mathrm{D}$	·3	7	124††	27·81	1008
$2l-l'-2\mathrm{D}$	·2	6	128††	8·01	1008
$3l+l'-3\mathrm{D}$	·03	4	50††	14·8	441
$3l-l'-3\mathrm{D}$	·03	3	42††	5·64	378
$l+2l'-\mathrm{D}$	·12	4	132††	30·81	1071
$l-2l'-\mathrm{D}$	·11	5	100††	9	819

19. *Remaining terms.*—The two terms $-0''·75\sin(2\mathrm{F}+2l-2\mathrm{D})$ $-0''·35\sin(2\mathrm{F}+l+l'-2\mathrm{D})$ present in S will probably be put into single-entry tables.

The terms $+0''·32\sin(2\mathrm{F}-l-l')-0''·32\sin(2\mathrm{F}-l+l')$ $=-0''·64\cos(2\mathrm{F}-l)\sin l'$ may be included by adding to the second argument of that double-entry table in S which has second argument $2\mathrm{F}-l-\mathrm{D}$, a term of the form $a\sin l'$; this term depends solely on the day of the year.

Two small terms with coefficients $\pm 0''{\cdot}003$ and arguments $F + l - 2D \pm 3l'$ are included by a similar addition to the second argument $F + l - 2D$.

A number of small planetary terms and one or two solar terms are included in two remainder tables with 10-day intervals.

There are small terms in S containing the argument 2F which will be omitted; the error thus caused can always be reduced to one-half of the coefficient of the omitted term. For a small term $a \sin(2F + \alpha)$ in S is the same as a term

$$ka \cos F \sin(2F + \alpha) = \tfrac{1}{2}ka \sin(F + \alpha) + \tfrac{1}{2}ka \sin(3F + \alpha)$$

in latitude. The term with argument $F + \alpha$ can be included by suitable additions to the coefficients of the terms with argument α in C, S. The omission will then be the term with argument $3F + \alpha$ having a coefficient one-half that of the omitted coefficient in kS.

The maximum error in latitude made by the omission of terms with coefficients greater than $0''{\cdot}002$ will be less than $0''{\cdot}06$.

20. *Comparison with Hansen.*—For the tabulation of the solar terms in latitude, Hansen has 18 double-entry and 10 single-entry tables; in addition, there is the table for the principal argument, and also one for the reduction of the sine-latitude to the latitude. I have 26 double-entry and 12 single-entry tables; in addition, a table for the principal argument, and the process of multiplication by the factor C. My tabulation, however, includes all terms greater than $0''{\cdot}01$, while Hansen omits 22 terms greater than this, the largest of them having coefficients $0''{\cdot}136$, $0''{\cdot}090$, $0''{\cdot}072$, $0''{\cdot}057$. With Hansen's degree of accuracy it would have been possible for me to omit 2 single-entry and 5 or 6 double-entry tables. Taking into account these and other differences, the methods may be fairly judged by the statement that I have 2 or 3 double-entry tables and one reduction of the sum of the double-entry tables from $D = 0$ to the value of D at date, to offset the tabulation of 18 arguments which in Hansen's method must be formed for the latitude, but which, in my method, have already been obtained for the longitude.

Hence, so far as the general plan is concerned, there is little choice between the two methods. When, however, the labour of extracting the numbers from the tables is considered—and this constitutes by far the longer part of the work—a greater difference will be noticed. In the new single-entry tables the divisions of the argument are much smaller, and therefore the labour of interpolation much less, and nearly all the arguments remain unchanged for a whole year. The use of the double-entry tables will be made easier by printing the variations per unit change of the argument, and by placing successive half-days in column rather than in line.

21. A large proportion of the planetary terms in latitude are accounted for by the fact that the inclusion of planetary terms in longitude through λ also takes account of certain classes of

planetary terms in latitude. The remainder of the terms can be included in two tables at 10-day intervals, one in C and one in S.

22. It is now possible to make a forecast of the number of pages of the size in Hansen's tables which will be required. Reckoning on the basis of 800 entries per page on the average for the single-entry tables, and 1000 per page for the double-entry tables, about 350 pages will be required for the terms exhibited in the seven foregoing tables. An estimate of 100 more pages is made for the remainder and planetary terms; 20 pages will be ample for the arguments. Making an allowance of 30 pages for loss of space in fitting the tables into pages, we arrive at 500 as the number required. I believe it will be safe to say that the number will not exceed 550 pages. These estimates are made with a somewhat smaller spacing between the columns than that adopted by Hansen: since the computer always follows columns, and never lines, this arrangement will not be objectionable. Hansen's tables occupy 437 pages. Hence the increase of size over Hansen is smaller than might be expected with the much greater number of terms and the much more minute divisions of the arguments of the single-entry tables. It is, of course, due to the new method of tabulation for the single-entry tables and the replacing of the values for quarter-days by the differences in the double-entry tables.

23. Frequent discussions on these plans between Mr. H. B. Hedrick and myself, and careful examination of them by him before final adoption, have been of great assistance. The construction of the forms for the actual computation has been mainly done by him. In the numerical work he has for the past twelve months been assisted by one computer, and since the beginning of the present year by two computers, who have given a portion of their time to the work.

In order to avoid computations in duplicate, careful checks are made at each stage of the work by means of differences. Errors which would not be revealed in this way are avoided by comparing a few values obtained at arbitrary dates from the finished table with values obtained directly from the terms contained in the table. Interpolation has been freely used in the work; and this in itself, when carried out in a proper manner, constitutes a valuable check on the accuracy of the results.

New Haven, Conn.:
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The Transformation of the Moon's Latitude.

By Ernest W. Brown, F.R.S.

1. In the *Memoirs* of the Society, vol. lix. pp. 136–141, the latitude of the Moon is obtained in the form

$$U = \Sigma u \sin(F + \alpha) + \Sigma u' \sin(3F + \alpha) + \Sigma u'' \sin(5F + \alpha), \quad . \quad (1)$$

where α is an argument formed of multiples of l, l', D, and u, u', u'' are coefficients expressed in seconds of arc.

It is desired, for purposes of tabulation, to transform this into

$$U = (k + C)\{\sin(F + S) + \mu \sin 3(F + S) + \nu \sin 5(F + S) + N\}, \quad (2)$$

where N consists of a few terms with large coefficients so chosen that the terms in C may all be put into double-entry tables. The additional conditions laid down are as follows: C is to contain no constant term, and to be independent of the argument F; μ, ν are so chosen as to account for the principal terms containing the third and fifth multiples of F in (1); S is to consist of periodic terms only, and, as far as possible, to include terms with large coefficients which have been placed in single-entry tables in longitude, so that there will not be any necessity for new single-entry tables for the remaining terms.

2. The problem of transformation from (1) to (2) is an indeterminate one until μ, ν, N have been chosen. Even when the choice has been made, there is no ready method of performing it. Processes of adjustment to find the most suitable values of μ, ν, N, and then of approximation to compute k, C, S, seem to be the only means available to obtain the best results. It is, therefore, not an easy matter to describe briefly the numerical work which was actually performed. It is further complicated by the fact that a transformation had previously been made into the form

$$U = (k + C)\sin(F + S) + N \quad . \quad . \quad . \quad (3)$$

with the idea that this was the best form for tabulation. Here N, in addition to certain large terms, contained the terms having in their arguments the third and fifth multiples of F; C was defined as before; and S was independent of the argument F. Both for purposes of verification and to save labour, it was desirable to use (3) as far as possible in obtaining (2).

The chief difficulty arises from terms with large coefficients which must necessarily be present in S, and which, on expansion of (2) or (3) in powers of S, will require the computation of powers up to S^5.

The expression (1) contains over 300 terms. Of these, the principal term has a coefficient 18461″, which is also nearly the value of k; six others have coefficients 1010″, 1000″, 623″, 199″, 166″, 117″, the first two of which give a coefficient of 1/9 in circular measure in S; seven coefficients range between 62″ and

10″, and thirty-one between 10″ and 1″. As the degree of accuracy in (1) is 0″·001, any coefficient over 15″ in S must have its square included, while every coefficient greater than 0″·01 will produce a portion greater than 0″·001 when S^2 is formed.

Some assistance is derived from an examination of the terms in Hansen's form

$$\sin U = k \sin (F + \lambda) + s,$$

where λ includes all the periodic terms in longitude independent of F, and s contains all other terms. It is also evident that if we have two large terms $a \sin (F + \alpha) - a' \sin (F - \alpha)$ in (1), where a, a' are nearly equal, they will give rise to a large term in S and a small one in C; while if a, a' are not nearly equal and one of them is large, we shall have a term to be placed in N in order that the term in C may not be large.

The general plan used for the transformation is described below. It will be understood, however, that no attempt is made to give full details, which are of no special interest in themselves, and which do not permit of a brief description.

3. The first step was the transformation of

$$\sin (F + S_0)$$

into a sum of sines. Here S_0 consisted of the following terms present in the true longitude :—

$$\begin{aligned}
S_0 = &+ 22639''\cdot580 \sin l &&- 124''\cdot785 \sin D &&+ 4586''\cdot477 \sin (2D - l)\\
&+ 769\cdot021 \sin 2l &&+ 2369\cdot909 \sin 2D &&+ 30\cdot773 \sin 2(2D - l)\\
&+ 36\cdot124 \sin 3l &&+ \cdot402 \sin 3D &&+ \cdot293 \sin 3(2D - l)\\
&+ 1\cdot938 \sin 4l &&+ 13\cdot902 \sin 4D &&+ \cdot004 \sin 4(2D - l)\\
&+ \cdot113 \sin 5l &&+ \cdot004 \sin 5D &&- 18\cdot554 \sin (D - l)\\
&+ \cdot007 \sin 6l &&+ \cdot127 \sin 6D &&+ 211\cdot658 \sin 2(D - l)\\
&&&&&- \cdot045 \sin 3(D - l)\\
&+ 191\cdot954 \sin (2D + l) &&+ 38\cdot428 \sin (4D - l) &&- \cdot003 \sin 4(D - l)
\end{aligned}$$

—which expression includes all the largest terms containing the arguments l, D only.

The method of harmonic synthesis and subsequent analysis was used. The function S_0 was computed with the values

$$\begin{aligned}
D &= 0^\circ, \pm 30^\circ, \pm 60^\circ, 90^\circ, \pm 120^\circ, \pm 150^\circ, 180^\circ,\\
l &= 0^\circ, \quad 30^\circ, \quad 60^\circ, 90^\circ, \quad 120^\circ, \quad 150^\circ, 180^\circ,
\end{aligned}$$

and $\cos S_0$, $\sin S_0$ were found for every pair of combinations of these values of D, l. They were then analysed into sums of cosines and sines of angles of the form $iD + jl$, the analysis being first made for the seven values of l, and then for the values of D.

The results were next put into the form $\Sigma a_{ij} \sin (F + iD + jl)$, and multiplied by a constant k_0 so chosen that $k_0 a_{00}$ was equal to the coefficient of sin F in (1).

This part of the work is necessary in any similar transformation,

and the numerical results are therefore given in the table of No. 4.

The method to a certain extent carries its own verification, since we know in advance that the high multiples of D, l must have small coefficients, but for additional security some checks were computed with multiples of the angle 45°.

4. *Transformation of* $18520''\cdot186\sin(F+S_0)=\Sigma s\sin(F+jl+iD)$.

j	i	s	j	i	s	j	i	s
		$''$			$''$			$''$
0	−6	− 0·004	2	−6	+ 0·022	4	−4	+ 0·008
	−4	− ·190		−4	− ·217		−2	− ·078
	−3	− ·046		−3	− ·016		0	+ ·265
	−2	− 95·245		−2	− 21·095		2	+ ·004
	−1	+ 5·576		−1	+ ·070			
	0	+ 18461·480		0	+ 61·960	−4	0	− ·011
	1	− 5·586		1	− ·018		2	− ·006
	2	+ 116·855		2	+ ·872		4	− ·004
	3	− ·018		4	+ ·009			
	4	+ 1·188						
	6	+ ·012	−2	−2	+ ·514	5	−2	− ·005
				−1	− ·002		0	+ ·018
1	−6	+ ·012		0	− 6·724			
	−4	− ·538		1	+ ·048	−5	0	− ·001
	−3	− ·068		2	− 1·884			
	−2	− 210·618		3	− ·008			
	−1	+ 1·200		4	+ 2·418			
	0	+ 1011·469		5	− ·001			
	1	− ·299		6	+ ·034			
	2	+ 15·088						
	3	− ·004	3	−4	+ ·094			
	4	+ ·116		−3	+ ·001			
	6	+ ·001		−2	− 1·234			
				−1	+ ·004			
−1	−4	+ ·062		0	+ 3·994			
	−2	− 2·678		1	− ·001			
	−1	− ·303		2	+ ·054			
	0	− 1017·933						
	1	− ·462	−3	−2	+ ·004			
	2	+ 199·916		0	− ·234			
	3	− ·065		2	− ·598			
	4	+ 2·988		3	− ·002			
	5	− ·001		4	− ·038			
	6	+ ·023		6	+ ·032			

5. The earlier computation of (3) and an approximate transformation of it into the form (2) having shown what further large terms, involving not only the angles l, D, but also the angles l', 2F, would have to be present in S, the expansion of

$$(k_0 + k_1) \sin (F + S_0 + S_1) \quad . \quad . \quad . \quad . \quad (4)$$

was performed. Here k_1 is the part additional to k_0 necessary in order that the coefficient of sin F in this expansion should be the same as in (1), and S_1 consisted of the following terms, the size of some of the coefficients having been estimated in the manner just indicated, and the others being those of terms in the longitude :—

$$\begin{aligned} S_1 = &- 28''\cdot700 \sin l &&- 126''\cdot200 \sin l' &&- 53''\cdot000 \sin (2F - 2D) \\ &+ 58\cdot900 \sin (2l - 2D) &&- 165\cdot351 \sin (l' - 2D) &&- 83\cdot000 \sin (2F - l) \\ &+ 14\cdot387 \sin (2l + 2D) &&- 24\cdot451 \sin (l' + 2D) \end{aligned}$$

$$\begin{aligned} &- 109''\cdot804 \sin (l + l') &&+ 147''\cdot878 \sin (l - l') \\ &- 182\cdot219 \sin (l + l' - 2D) &&+ 28\cdot511 \sin (l - l' - 2D) \\ & &&+ 14\cdot595 \sin (l - l' + 2D) \end{aligned}$$

The computation was made in the form

$$\cos (S_0 + S_1) - \cos S_0 = - \tfrac{1}{2} S_1^2 - S_1 \sin S_0$$
$$\sin (S_0 + S_1) - \sin S_0 = S_1 - \tfrac{1}{2} S_1^2 \sin S_0 + S_1 (\cos S_0 - 1).$$

Since sin S_0, cos S_0 have been already found as sums of sines and cosines, the right-hand members were computed by forming the products of the series. The terms in the expansion beyond those set down are insensible.

6. The expanded value of (4) thus obtained was subtracted from (1). The large terms remaining were set aside, and constituted the principal part of the value of N. In (2), N is multiplied by C. The approximate value C_0 of C was already known, and the values to be given to N and S had been adjusted so as to leave the terms in C as small as possible. A similar process gave the values of μ, ν. Denote the form (1) by U. The computations undertaken up to this point gave the value of

$$\begin{aligned} U - (k_0 + k_1) \{ &\sin (F + S_0 + S_1) + \mu \sin 3(F + S_0 + S_1) \\ &+ \nu \sin 5(F + S_0 + S_1) + N \} - C_0 N \quad . \quad (5) \end{aligned}$$

and this expression then contained only small terms. In order to include these, a variable part C is to be added to k_0 and a portion S_2 to S. If it were possible to neglect squares and products of

μ, ν, C, S_2, and the product $(C - C_0)N$, these terms would be expressible in the form

$$C \sin (F + S_0 + S_1) + \{k_0 + k_1\} S_2 \cos (F + S_0 + S_1), \quad . \quad (6)$$

where C is added to $k_0 + k_1$, and S_2 to $F + S_0 + S_1$.

There were, however, a number of them producing terms in the latitude with coefficients lying between 0″·000 and 0″·010; these were treated as corrections to C, S_2 after these two quantities had been obtained.

7. The only step remaining to be performed was therefore the determination of C, S_2. Denote the terms in (5) by

$$\Sigma u \sin (F + \alpha) + \Sigma u' \sin (3F + \alpha), \quad . \quad . \quad . \quad (7)$$

there being none left with the multiple 5F. The condition laid down was that C should not contain the argument F.

Let

$$C = \Sigma c \cos \alpha, \quad S_2 = \Sigma s \sin \alpha + 2\Sigma s' \sin (2F + \alpha) \quad . \quad (8)$$

where c, s, s' are coefficients and α the corresponding argument containing multiples of l, D, l' only.

Substituting (8) in (6), and equating the terms containing multiples of 3F, F to those in (7) separately, we obtain

$$(k_0 + k_1)\Sigma s' \sin (3F + \alpha + S_0 + S_1) = \Sigma u' \sin (3F + \alpha),$$
$$(\Sigma c \cos \alpha) \sin (F + S_0 + S_1) + (k_0 + k_1)(\Sigma s \sin \alpha) \cos (F + S_0 + S_1)$$
$$+ (k_0 + k_1)\{\Sigma s' \sin (F + \alpha - S_0 - S_1)\} = \Sigma u \sin (F + \alpha),$$

for all values of F.

From these we immediately deduce

$$(k_0 + k_1)(\Sigma s' e^{\alpha\iota}) = (\Sigma u' e^{\alpha\iota}) e^{-S_0\iota - S_1\iota}$$
$$\Sigma\{c + (k_0 + k_1)s\iota\} e^{\alpha\iota} = (\Sigma u e^{\alpha\iota}) e^{-S_0\iota - S_1\iota} - \{\Sigma(k_0 + k_1) s' e^{\alpha\iota}\} e^{-2S_0\iota - 2S_1\iota}$$
$$= (\Sigma u e^{\alpha\iota}) e^{-S_0\iota - S_1\iota} - (\Sigma u' e^{\alpha\iota}) e^{-3S_0\iota - 3S_1\iota}.$$

Since we know $e^{S_0\iota - S_1\iota} = \cos (S_0 + S_1) - \iota \sin (S_0 + S_1)$, and from this can compute $\cos 3(S_0 + S_1) - \iota \sin 3(S_0 + S_1)$ as sums of sines and cosines, the right-hand members of the two equations can be obtained by multiplication of series. These multiplications were performed with exponentials (instead of with sines and cosines) in the manner used so frequently in my lunar theory. All the coefficients c, s, s' were thus obtained.

8. Verifications of these steps were made by finding the sums of the coefficients for each multiple of l, l', 2F and all multiples of D. After the final results of the transformation had been obtained, some checks on the whole were computed with the

angles 0, 45°, 90°, so substituted for l, l', D, 2F as to form as efficient a test as possible. The largest accumulated error obtained was less than 0″·02, and this was partly accounted for by a number of minute terms less than 0″·002, which were rejected since they would not be tabulated in any case.

9. *Final results.*—The latitude is expressed in the form

$$(k+\mathrm{C})\{\sin(\mathrm{F}+\mathrm{S})+\mu\sin 3(\mathrm{F}+\mathrm{S})+\nu\sin 5(\mathrm{F}+\mathrm{S})+\mathrm{N}\}$$

with the coefficients expressed in seconds of arc. Since the terms in S are to be multiplied by $k=\frac{1}{11}$, they are carried only to hundredths of a second of arc; the rest are given to thousandths.

The values are as follows:—

$$k=18519''\cdot830,\quad k\mu=-6''\cdot240,\quad k\nu=+0''\cdot004,$$

$$\begin{aligned}
k_0\mathrm{N}=&-526''\cdot100\sin(\mathrm{F}-2\mathrm{D}) && -30''\cdot600\sin(\mathrm{F}-l-2\mathrm{D})\\
&-\ 3\cdot350\sin(\mathrm{F}-4\mathrm{D}) && -24\cdot650\sin(\mathrm{F}-2l)\\
&+\ 44\cdot300\sin(\mathrm{F}+l-2\mathrm{D}) && -\ 2\cdot000\sin(\mathrm{F}-2l-2\mathrm{D})\\
&-\ 6\cdot000\sin(\mathrm{F}+l-4\mathrm{D}) && -22\cdot600\sin(\mathrm{F}+l'-2\mathrm{D})\\
&+\ 20\cdot600\sin(\mathrm{F}-l) && +11\cdot000\sin(\mathrm{F}-l'-2\mathrm{D})
\end{aligned}$$

where $k_0=18519''\cdot600$.

Let $\mathrm{C}=\Sigma c\cos(i\mathrm{D}+jl+j'l')$, $\mathrm{S}=\Sigma s\sin(i\mathrm{D}+jl+j'l')+\Sigma s'\sin(2\mathrm{F}+i\mathrm{D}+jl+j'l')$.

The values of c, s, s' are given in the following table.

j	j'	i		s	c
				"	"
0	0	1	−	112·47	−0·723
		2	+	2373·37	+ ·601
		3	−	4·00	+ ·393
		4	+	14·06	− ·001
		5	−	·13	+ ·012
		6	+	·60	− ·042
		8	+	·01	− ·001
1	0	−8	−	0·03	−0·003
		−6	−	1·43	− ·092
		−5	+	·25	+ ·020
		−4	−	38·64	+ ·001
		−3	+	5·42	+ ·191
		−2	−	4578·18	− ·077
		−1	+	3·58	− ·094
		0	+	22609·15	+ ·079
		1	−	13·47	+ ·454
		2	+	192·72	+ ·029
		3	−	·74	+ ·068
		4	+	6·98	− ·445
		5	−	·01	+ ·001
		6	+	·25	− ·018
2	0	−8	−	·07	− ·004
		−6	−	1·40	− ·074
		−5	+	·12	+ ·007
		−4	−	34·07	− ·308
		−3	+	·91	− ·030
		−2	−	152·53	+5·679
		−1	+	2·00	− ·018
		0	+	767·96	+ ·107
		1	−	1·20	+ ·054
		2	+	14·78	− ·017
		3	−	·10	+ ·007
		4	+	1·02	− ·074
		6	+	·03	− ·003
3	0	−8	−	0·01	−0·001
		−6	−	·31	− ·002
		−5	+	·03	− ·001
		−4	−	·74	+ ·042
		−3	+	·05	− ·002
		−2	−	16·40	+ ·258
		−1	+	·19	− ·005
		0	+	50·64	−1·300
		1	−	·09	+ ·006
		2	+	2·96	− ·166
		4	+	·16	− ·012
4	0	−6	−	·03	− ·001
		−4	+	·02	
		−2	−	1·58	+ ·052
		0	+	3·60	− ·145
		2	+	·30	− ·023
5	0	−4	+	·01	− ·001
		−2	−	·14	+ ·005
		0	+	·28	− ·015
		2	+	·04	− ·002
0	1	−6	−	·22	− ·016
		−4	−	6·47	− ·417
		−3	+	·28	+ ·031
		−2	−	165·26	+ ·054
		−1	+	·32	− ·001
		0	−	127·14	−1·304
		1	+	17·90	+ ·007
		2	−	25·12	+ ·040
		3	+	·53	− ·032
		4	−	1·59	+ ·123
		5	+	·01	− ·001
		6	−	·06	+ ·005

j	j'	i		s		c
0	2	−4	−	0″·65	−	0″·044
		−3	+	·01	+	·001
		−2	−	16·39	−	·742
		−1	−	·04		
		0	−	·66	−	·037
		1	−	·04		
		2	−	1·68	+	·131
		4	−	·04	+	·004
0	3	−4	−	·01	−	·001
		−2	−	·57	−	·025
1	1	−6	−	·37	−	·027
		−5	+	·01	+	·001
		−4	−	9·67	−	·472
		−3	+	·36	+	·012
		−2	−	182·59	+	2·059
		−1	−	·12	+	·005
		0	−	115·35	+	·462
		1	+	1·52	−	·022
		2	−	11·76	+	·788
		3	+	·08	−	·007
		4	−	·50	+	·041
		6	−	·01	+	·001
2	2	−6	−	·02		
		−4	−	·16	+	·002
		−2	−	·27	+	·002
		0	−	·09	+	·002
1	−1	−6	+	·09	+	·006
		−5	−	·01	−	·001
		−4	+	2·27	+	·146
		−3	−	·38	−	·006
		−2	+	23·57	−	·444
		−1	+	·55	+	·021

j	j'	i		s		c
1	−1	0	+	138″·90	+	0″·680
		1	−	·33	+	·016
		2	+	31·84	−	1·542
		3	−	·04	+	·004
		4	+	1·53	−	·111
		6	+	·06	−	·005
2	−2	−4	+	·04		
		−2	+	·21	−	·003
		0	+	·22		
		2	+	·20	−	·010
2	1	−6	−	·25	−	·014
		−4	−	2·54	+	·022
		−3	+	·07	−	·002
		−2	−	7·60	+	·078
		−1	+	·02		
		0	−	10·57	+	·259
		1	+	·14	−	·003
		2	−	1·45	+	·116
		4	−	·07	+	·006
2	−1	−6	+	·06	+	·003
		−4	+	·20	−	·012
		−3	+	·04		
		−2	−	1·17	−	·003
		−1	−	·37	+	·001
		0	+	11·68	−	·151
		1	−	·04		
		2	+	3·32	−	·212
		4	+	·22	−	·018

j	j'	i	s	c
			"	"
3	1	−6	−0·06	−0·001
		−4	− ·08	+ ·003
		−2	− ·57	+ ·005
		0	− ·94	+ ·032
		1	− ·01	− ·001
		2	− ·17	+ ·011
3	−1	−6	+ ·01	− ·001
		−2	− ·23	+ ·003
		−1		+ ·002
		0	+ ·96	− ·026
		2	+ ·36	− ·022
		4	+ ·01	− ·001
4	1	−2	+ ·02	+ ·006
		0	− ·01	+ ·007
4	−1	−2	+ ·01	+ ·003
		0	− ·02	− ·008
		2	+ ·02	− ·002
1	2	−6	− ·03	− ·001
		−4	− ·65	− ·032
		−2	−6·14	+ ·117
		0	−1·25	+ ·008
		2	− ·13	+ ·014
1	−2	−4	+ ·07	+ ·006
		−2	+2·41	− ·014
		0	+2·33	+ ·027
		2	+1·82	− ·105
		4	+ ·12	− ·009

j	j'	i	s	c
			"	"
1	3	−4	− 0·02	
		−2	− ·22	+0·004
		0	− ·02	
1	−3	0	+ ·04	
		2	+ ·06	− ·003
			s'	
0	0	−6	− 0·03	
		−4	− 1·67	
		−3	+ ·25	
		−2	−52·14	
		−1	+ ·84	
		0	− ·20	
		2	− ·04	
		4	− ·02	
1	0	−6	− ·04	
		−5	− ·02	
		−4	− ·33	
		−3	+ ·04	
		−2	− 9·52	
		−1	+ ·07	
		0	− ·02	
		2	− ·02	
−1	0	−4	+ ·04	
		−3	− ·01	
		−2	+ 3·37	
		−1	+ ·04	
		0	−85·13	
		1	+ ·06	
		2	− ·71	
		3	+ ·02	
		4	− ·02	

j	*j'*	*i*	*s'*
2	0	−4	+0″·02
		−2	− ·75
		−1	− ·01
−2	0	−4	+ ·02
		−2	+ ·38
		0	− ·74
		2	−1·14
		4	− ·02
3	0	−2	− ·07
		0	− ·04
		2	− ·02
−3	0	−2	+ ·04
		0	− ·11
		2	+ ·02
		4	− ·08
4	0	−2	+ ·01
		0	+ ·02
−4	0	0	− ·02
0	1	−4	− ·17
		−3	+ ·02
		−2	−2·26
		0	+ ·10
		2	− ·02
0	−1	−4	+ ·08
		−2	+1·30
		−1	− ·06
		0	+ ·16
		2	+ ·04
0	2	−4	−0″·02
		−2	− ·09
	0	−2	−2	+ ·02
		2	+ ·02
1	1	−4	− ·03
		−2	− ·35
		0	+ ·02
−1	−1	−2	− ·02
		0	+ ·31
		2	− ·07
1	−1	−4	+ ·01
		−2	+ ·07
		0	+ ·03
		2	+ ·02
−1	1	−4	− ·01
		−2	+ ·19
		0	− ·33
2	1	0	− ·02
−2	−1	0	+ ·02
		2	+ ·02
2	−1	−2	− ·01
		0	+ ·02
−2	1	−2	+ ·01
		0	− ·02

New Haven, Conn.:
1911 May 6.

A Reply to Professor Ginzel on the Calendar Dates in the Elephantine Papyri. By J. K. Fotheringham, Litt.D.

Professor Ginzel, in the second volume of his Chronological Manual,* discusses the calendar dates in the Elephantine papyri, which had been previously discussed by Mr. Knobel and myself in *Monthly Notices*, lxviii. 334–345; lxix. 12–20. The section devoted by Professor Ginzel to this subject is an expansion, with full references to the literature of the subject, of Schürer's article, *Der jüdische Kalender nach den aramäischen Papyri von Assuan*,† for which Professor Ginzel himself had supplied the chronological material. Professor Ginzel and I are in general agreement about the identification of the dates found in these papyri and about the irregularity of the intercalation, though he does not mention my correction of two days in the date of papyrus E and the date which I assign to papyrus G, which he had left undated. We differ, however, in our opinions about the method by which the beginning of each Jewish month was determined. According to Professor Ginzel, this was obtained by observation of the lunar crescent; while I hold that it was obtained by calculation, each month beginning at the sunset following the mean new moon.

Professor Ginzel estimates that if we assume that the Moon made its first appearance at the age of $1\frac{1}{4}$ or $1\frac{1}{2}$ days, the dates of first appearance will satisfy the requirements of the papyri; and in order to exemplify this, he gives a list of dates of the astronomical new moons in question, expressed in Aswan mean time reckoned from noon, and also of the assumed dates of the first appearance of the crescent. In order to make the relation of the two more clear, I have substituted for the latter the times of the sunsets at Aswan at which the different calendar months are shown by the papyri to have begun, *i.e.* the time of the sunset at the beginning of the first day of each month, and I have added the resultant interval between new moon and this initial sunset. I have added to the list the dates which I have obtained from papyrus G. I thus obtain—

	Date of New Moon.	Date of Initial Sunset.	Difference, Sunset − New Moon. d
A	−470 August 24·28	August 25·27	0·99
B	−464 December 14·54	December 15·22	0·68
D	−459 October 20·59	October 21·23	0·64
G	−445 September 17·27	September 18·25	0·98
E	−445 November 15·75	November 14·22	−1·53
F	−439 August 12·31	August 12·27	−0·04
J	−415 December 12·48	December 13·22	0·74
K	−409 January 16·63	January 17·23	0·60

* *Handbuch der mathematischen und technischen Chronologie*, ii. (1911), pp. 45–52.

† *Theologische Literaturzeitung*, Feb. 2, 1907.

As I mentioned in my previous paper, the date of E is not only unique in implying a calendar month beginning at the second sunset before new moon, but is inconsistent with the date of G, which suggests an error of two days in the date of E. If E is omitted from consideration, the calendar months will be seen to begin, with one exception, at the sunset following the astronomical new moon, but the interval falls far short of the $1\frac{1}{4}$ or $1\frac{1}{2}$ days which Professor Ginzel desiderates. The question, therefore, arises whether the Moon could possibly have been seen at the comparatively short intervals after conjunction, resulting from the above table. In order to test this, I have availed myself of the rule which I published in *Monthly Notices*, lxx. 530, and which Professor Ginzel has reprinted in his Manual,* and have computed the true altitude of the Moon and the true difference in azimuth of Sun and Moon at Elephantine for each of the sunsets above, except that of the erroneously dated papyrus E. Professor Ginzel remarks † that the result of my rule may be essentially modified by atmospheric conditions, and that the lunar places obtained from our tables for distant dates are only vaguely approximate. I think I may estimate the maximum error in my altitudes, resulting from the latter cause, at $\pm 0^\circ{\cdot}2$. I find in this way—

	Difference in Azimuth, Sun – Moon.	Altitude of Moon.	Altitude required to render Moon visible.
	°	°	°
A	6·8	10·1	11·7
B	3·1	9·2	11·9
D	2·8	8·7	11·9
G	11·7	5·2	11·3
F	− 0·7	− 0·1	12·0
J	8·3	4·5	11·6
K	0·4	8·5	12·0

The difference between the figures in the second and third columns of this table is so striking that, when all allowance is made for the possibility of the air at Elephantine being clearer than at Athens, at which the observations which I used were made, and for the possibility of the Moon being occasionally seen at a lower altitude than would normally be possible in fair weather, it remains evident that in all or nearly all these instances the Moon would not be visible on the evening of the sunset with which the calendar month began.

I feel bound, therefore, to reject Professor Ginzel's opinion that the beginning of the month was determined by observation of the lunar crescent, and to hold to the view which I formerly expressed, that strict calendar rules were employed which aimed at making

* ii. 318. † *Ubi supra.*

each month begin at the sunset following mean new moon. As I stated in my former article, the dates in the papyri before us imply a value for the mean lunation of not less than $29^d\ 12^h\ 43^m\ 44^s{\cdot}63$, and not more than $29^d\ 12^h\ 44^m\ 51^s{\cdot}15$.*

Magdalen College, Oxford:
1911 *June* 8.

On the Hartmann-Cornu Formula for the Reduction of Spectrograms. By F. J. M. Stratton, M.A.

§ 1. *The Hartmann-Cornu Formula.*—The method in most general use for the reduction of prismatic spectrograms is the very convenient one developed by Dr. Hartmann in No. 42 of the *Publicationen des Astrophysikalischen Observatoriums zu Potsdam* (appendix to vol. xii.). If n is the measured scale-reading of a line of wave-length λ, and if we plot the points (λ, n) corresponding to a given spectrogram, then the points obtained lie on a curve which can be very well represented by the formula

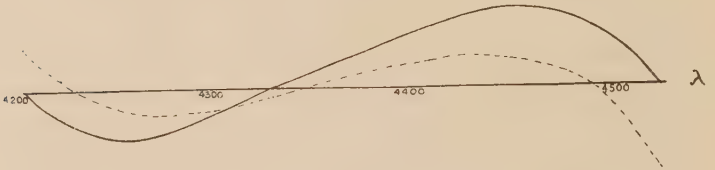
$$n - n_0 = \frac{c}{(\lambda - \lambda_0)^\alpha} \quad . \quad . \quad . \quad . \quad (1)$$

where α is a constant depending on the spectrograph employed; n_0, c, λ_0 are constants determined for each plate by making the curve pass through three of the plotted points, *i.e.* by using three of the measured lines as standards.

§ 2. *Proposed modification of the method.*—This paper extends the use of the above formula so as to admit of more than three lines being used as standards in the reduction. The growing number of lines of accurately known wave-lengths, based on the international system, renders possible a wider choice of suitable standard lines in a comparison spectrum to a star photograph, while the use of a one-line comparison spectrum in the determination of radial velocities by an objective prism makes it desirable to find some method of using more than three star lines in order that sufficient accuracy may be obtained. Another reason for using a large number of lines in the reduction lies in the systematic difference found by Hartmann † and Newall in the behaviour of faint and and strong lines. Professor Newall suggests, as a probable explanation of this difference, varying refrangibility across the prism, which affects the strong lines due to light coming through the whole of the prism. In a star spectrum the lines measured frequently differ considerably in density. Errors arising from this fact might be to some extent smoothed out if numerous comparison lines could be chosen of varying density.

* See my paper in *Monthly Notices*, lxix. 19, and erratum.
† *Ast. Nach.*, Bd. 155, 93.

§ 3. *Advantages of the method.*—Over the greater part of the range of spectrum under examination a closer fit is obtained by the use of an increased number of standard lines in determining the constants in curve (1). The residuals between the observed and computed positions of lines are found to be slightly increased at the ends of the range. The two curves in the diagram show the distribution of residuals in a given case when the two methods of reduction were used. The continuous curve corresponds to an ordinary three-line reduction, the dotted curve to a reduction with twenty-five lines as standards; the range of spectrum considered is 4202λ to 4529λ.



The sum of the squares of the residuals was decreased from 0·391 to 0·147 by using the additional number of standard lines. These figures give a somewhat too flattering view of the success of the new method in improving the fit in general. As will be seen, the residuals are systematic in their arrangement, and this can be allowed for partly by an empirical correction along the range—a correction derived from a group of carefully measured suitable plates. Or, again, the systematic distribution of the residuals can be greatly lessened by a careful choice of α, the spectrograph-constant of (1). To remove it entirely, however, involves considerable labour; and there will always be, even with the best value of α, or the best curve of empirical corrections, improvement possible in securing agreement between observation and the curve of fit. In every case to which it has been applied, the method outlined below does give a closer fit than the ordinary three-line reduction. It has the added advantage that the chance of accidental errors in the measurement of any standard line affecting the reduction is much diminished when the number of standard lines is increased. The extra labour involved is almost entirely preliminary in nature. Once certain constants, which are the same for a series of plates taken with the same instrument for the same range of spectrum, have been obtained, the work involved in the reduction of each plate is no more than in the ordinary three-line reduction.

§ 4. *The Constants of the Formula.*—We shall deal first with Hartmann's simple formula with $\alpha = 1$,

$$n - n_0 = \frac{c}{\lambda - \lambda_0} \quad . \quad . \quad . \quad . \quad (2)$$

This curve for (λ, n) is a rectangular hyperbola. The constant n_0

depends upon the position in which the plate is placed in the measuring machine, and is liable to vary fairly widely from plate to plate. The constant c depends upon the scale value of the plate; it varies with the temperature and with the adjustment of the spectrograph. The constant λ_0 is approximately fixed for a series of plates taken by the same instrument for the same range of spectrum. If λ_0 is determined for each plate separately by a three-line reduction it is found to vary slightly from plate to plate, but there is very little loss in exactness of fit for the whole range if an approximate value of λ_0 is fixed and used for all the plates of the series. Satisfactory accuracy can be obtained by proper choice of n_0 and c. The closeness of fit of the hyperbola need only be slightly affected if one of its asymptotes is fixed in position, so long as the other asymptote is free to move and the size of the axis is allowed to vary.

§ 5. *The use of more than three Standards.*—In the hyperbola (2)

$$(n - n_0)(\lambda - \lambda_0) = c$$

let us assume that a preliminary reduction by Hartmann's three-line method has given approximate values of c, λ_0, n_0. Corresponding to any line of known wave-length λ_r, the observed scale-reading n_r should be given by

$$n_r = n_0 + \frac{c}{\lambda_r - \lambda_0} \quad . \quad . \quad . \quad . \quad (3)$$

If equation (3) is not satisfied, let

$$\delta n_r = n_r - \left(n_0 + \frac{c}{\lambda_r - \lambda_0}\right) \quad . \quad . \quad . \quad (4)$$

We desire to find small corrections δn_0, δc, $\delta\lambda_0$ to our constants so as to satisfy equation (3).

The equations to be satisfied for the k standard lines of known wave-length λ_r are

$$n_r = (n_0 + \delta n_0) + \frac{c + \delta c}{\lambda_r - (\lambda_0 + \delta\lambda_0)} \qquad (r = 1, 2, \ldots k),$$

or from (4) on expanding

$$\delta n_r = \delta n_0 + \frac{\delta c}{\lambda_r - \lambda_0} + \frac{c\delta\lambda_0}{(\lambda_r - \lambda_0)^2} \qquad (r = 1, 2 \ldots k) \quad . \quad (5)$$

The method of least squares is used to determine the best values of δn_0, δc, $\delta\lambda_0$. In practice it is found that, if all three constants are allowed to vary, the weights of the derived corrections to them are very small, *i.e.* the corrections are very badly determined. But if λ_0 is assumed constant, and therefore $\delta\lambda_0 = 0$, then the resulting equations

$$\delta n_0 + \frac{\delta c}{\lambda_r - \lambda_0} - \delta n_r = 0 \qquad (r = 1, 2 \ldots k) \quad . \quad . \quad (6)$$

lead to a satisfactory hyperbola, which fits the observations more closely than the original hyperbola found from the three-line reduction.

§ 6. *The work involved in the Reduction.*—For a series of plates of the kind described above we have first to choose the list of k lines of known wave-length to be adopted as standards. For each line we calculate $\left(n_0 + \frac{c}{\lambda_r - \lambda_0}\right)$ and $\frac{1}{\lambda_r - \lambda_0}$, the constants which figure in equations (4) and (6). We also compute $\sum_1^k \frac{1}{\lambda_r - \lambda_0}$, and $\sum_1^k \frac{1}{(\lambda_r - \lambda_0)^2}$, two coefficients in the normal equations. After this preliminary work all that is left to do for each individual plate is the following:—

(*a*) Find for each line $\delta n_r = n_r - \left(n_0 + \frac{c}{\lambda_r - \lambda_0}\right)$: as the zero for the scale reading for any plate is arbitrary we may alter n_r and δn_r by a constant quantity throughout so that δn_r may be small. Find also $\sum_1^k \delta n_r$.

(*b*) Calculate for each line $\frac{\delta n_r}{\lambda_r - \lambda_0}$ and find $\sum_1^k \frac{\delta n_r}{\lambda_r - \lambda_0}$.

(*c*) Solve the normal equations

$$\left.\begin{aligned} k\delta n_0 + \sum_1^k \frac{1}{\lambda_r - \lambda_0} \cdot \delta c - \sum_1^k \delta n_r = 0 \\ \sum_1^k \frac{1}{\lambda_r - \lambda_0} \cdot \delta n_0 + \sum_1^k \frac{1}{(\lambda_r - \lambda_0)^2} \cdot \delta c - \sum_1^k \frac{\delta n_r}{(\lambda_r - \lambda_0)} = 0 \end{aligned}\right\} \quad . \quad (7)$$

and find δn_0, δc. We note that only the last term in each equation differs from plate to plate.

(*d*) Having obtained corrected constants $n_0 + \delta n_0$, $c + \delta c$, λ_0 for our curve, the reduction of the spectrogram is completed in the usual way.

§ 7. *Hartmann's more general Formula.*—The above method may be applied with equal ease when the restriction that $\alpha = 1{\cdot}0$ is removed, and the value of α best fitted to the spectrograph in use is taken. This value is derived by the method described by Hartmann * in the course of preliminary work on one or two plates, and at the same time approximate values of λ_0, n_0, c are obtained. We have now, as before, to compute for each standard line

$$n_0 + \frac{c}{(\lambda_r - \lambda_0)^\alpha}, \quad \frac{1}{(\lambda_r - \lambda_0)^\alpha}, \quad \frac{1}{(\lambda_r - \lambda_0)^{2\alpha}}; \quad \text{also} \quad \sum_1^k \frac{1}{(\lambda_r - \lambda_0)^\alpha} \quad \text{and} \quad \sum_1^k \frac{1}{(\lambda_r - \lambda_0)^{2\alpha}}.$$

* *Potsdam Publications*, No. 42, p. 7; *Ast. Nach.*, Bd. 155, 102.

For each plate we have to compute

$$(a) \qquad \delta n_r = n_r - \left(n_0 + \frac{c}{(\lambda_r - \lambda_0)^\alpha}\right), \text{ and } \sum_1^k \delta n_r,$$

$$(b) \qquad \frac{\delta n_r}{(\lambda_r - \lambda_0)^\alpha} \text{ and } \sum_1^k \frac{\delta n_r}{(\lambda_r - \lambda_0)^\alpha}.$$

and we have to solve the normal equations

$$\left.\begin{aligned} k\delta n_0 + \sum_1^k \frac{1}{(\lambda_r - \lambda_0)^\alpha} \cdot \delta c - \sum_1^k \delta n_r = 0 \\ \sum_1^k \frac{1}{(\lambda_r - \lambda_0)^\alpha} \cdot \delta n_0 + \sum_1^k \frac{1}{(\lambda_r - \lambda_0)^{2\alpha}} \cdot \delta c - \sum_1^k \frac{\delta n_r}{(\lambda_r - \lambda_0)^\alpha} = 0 \end{aligned}\right\} . \quad (8)$$

The solution gives the constants $n_0 + \delta n_0$, $c + \delta c$, λ_0 to be used in the reduction curve.

§ 8. *The Values of* α.—It may be of interest to add here the values of α which an examination of measures published by other observers and of Cambridge plates has shown to be best suited to various spectrographs.

Spectrograph.	Material of Prism.	Best Value of α.	Reference.
Potsdam, one prism	{ Flint glass / Crown glass }	1·2	*Potsdam Publications*, No. 42.
Cambridge, one prism	Mantois flint	0·8	
Cambridge, two prisms	Jena dense flint	0·6	
Lick, three prisms	Extra dense flint	0·6	*Astrophysical Jour.*, vol. 8, p. 143.
Potsdam, three prisms	White Jena dense flint	0·6	*Ast. Nac.*, 155, 103.
Cambridge, four prisms	Jena dense flint	0·25	

Some attempt has been made to connect the value of α with the optical properties of the various instruments. The exact significance of α has not, however, been traced. The range of spectrum covered in the above was from 4200λ to 4530λ for Cambridge and Potsdam, but only 4240λ to 4440λ for Lick.

§ 9. *Practical test of the method.*—As a first test of the usefulness of the method of reduction outlined in this paper, a group of five plates of α Boötis, which have already been carefully discussed by Professor Newall * were remeasured with more iron standard lines by Mr. W. H. Manning, and reduced by him and myself. Twelve standard iron lines and ten star lines were measured on each plate in the range 4202λ to 4326λ. Their wavelengths are given in the tables below: the standard iron lines all figure in the list of Tertiary Standards of the international system. The plates were reduced in three separate ways:—

(1) By the Hartmann three-line method with $\alpha = 1{\cdot}0$.

(2) With twelve lines as standards and $\alpha = 1{\cdot}0$, the values of

* *M.N., R.A.S.*, vol. lxiii. p. 299.

λ_0, n_0, c being derived from the three-line reduction of the first plate, F 334.

(3) With twelve lines as standards and $a = 0{\cdot}25$, the same plate as before serving to give the values of λ_0, n_0, c.

The object of this repetition of work was to examine the nature and size of the systematic differences between the various reductions, and also their relative success in giving a close-fitting curve. Over so short a range of spectrum the difference between the two reductions with $a = 1{\cdot}0$ and $a = 0{\cdot}25$ was found to be quite slight; the really important difference lay between the three-line and the twelve-line reductions. Here the advantage lay with the twelve-line reduction slightly as regards the size of the residuals, but strongly when their distribution was also taken into account.

§ 10. *Residuals of the Standard Lines.*—In the table below are given the wave-lengths of the standard lines and the residuals obtained in the various reductions. It will be noticed that one or two lines show a peculiar divergence from any smooth curve which could be drawn to represent the systematic misfit of the computed wave-lengths. Some such explanation as that developed in § 2 would probably account for these individual differences.

Residuals of Iron Lines $\lambda_{IA} - \lambda_C$*—Unit* 0·001Å.

Reduction	$a=0{\cdot}25$, 12 Lines.						$a=1{\cdot}0$, 12 Lines.						$a=1{\cdot}0$, 3 Lines.					
Plate No.	334	336	337	338	341	Mean.	334	336	337	338	341	Mean.	334	336	337	338	341	Mean.
Line																		
4202·030	− 3	− 7	+ 2	+18	+ 9	+ 4	+ 3	− 1	+ 6	+26	+17	+10	0	0	0	0	+ 1	0
10·357	−23	− 5	−29	− 8	− 9	−15	−24	− 1	−27	− 4	− 5	−12	−27	−35	−37	−27	−20	−29
19·365	+ 9	− 5	+ 9	+ 3	−11	+ 1	+ 7	− 6	+ 7	+ 4	−10	0	+ 4	−13	− 5	−14	−24	−10
22·220	0	−19	+17	−24	−15	− 8	− 2	−20	+16	−24	−15	− 9	− 6	−28	+ 2	−41	−29	−20
27·440	+ 9	+ 9	− 6	−11	+ 2	+ 1	+ 7	+ 7	− 8	−11	+ 1	− 1	+ 3	− 2	−22	−27	− 2	−10
50·789	+11	+ 5	+ 2	+ 1	− 3	+ 3	+11	+ 6	+ 2	+ 2	− 1	+ 4	+ 5	− 8	−16	− 7	− 9	− 7
60·484	+ 5	+11	+17	+ 5	+ 4	+ 8	+ 7	+13	+18	+ 8	+ 6	+10	0	0	0	0	+ 1	0
82·404	+ 7	+11	+13	+14	+24	+14	+14	+17	+18	+23	+31	+21	+ 5	+ 7	+ 3	+14	+33	+12
94·124	− 9	+ 5	− 8	− 8	− 1	− 4	− 3	+13	− 2	0	+ 7	+ 3	−13	+ 6	+16	−10	+13	+ 2
99·243	−22	−20	−17	−32	+ 2	−18	−15	−12	−11	−23	+11	−10	−25	−18	+23	−34	+17	− 7
4315·089	−16	−16	− 9	−14	− 9	−13	−13	−13	− 8	−11	− 7	−10	−24	−11	+15	−26	+ 9	− 7
25·763	+16	− 6	+ 7	+23	−15	+ 5	+12	− 9	+ 2	+19	−20	+ 1	0	0	0	0	+ 1	0
Mean sq. residual	13	11	14	16	11	10	12	11	13	16	14	10	14	15	16	21	17	12

An examination of the above figures shows that, as was to be expected, each reduction fits worst over some part of the range and best over another part. The mean square residual, in so far as it is a satisfactory test, shows an advantage in the twelve-line reductions. As eight of the ten star lines measured lie within the range 4203–4252, we should, from the above figures, expect the most trustworthy resultant velocities from one of the twelve-line reductions and a large systematic error in the three-line reductions. As we shall see in the next paragraph, a large systematic difference does reveal itself between the velocities obtained by the twelve-line and three-line reductions.

§ 11. *Resultant Velocities.*—There are two difficulties in determining star velocities from the measures of spectrograms which call for mention here. The identification of the lines in the star photograph with individual lines or blends of lines in Rowland's tables is the first task. In this case it was decided to adopt the same Rowland lines as those used by Professor Newall in the earlier paper.* The second difficulty lies in reconciling the wave-lengths of the international system with those of Rowland. Professor Kayser † recommends the subtraction of 0·19λ from all measurements on the Rowland system to reduce to the I.A. system. The residuals given in Kayser's table show for the lines in the range of spectrum under examination a mean difference between the two systems of 0·160λ, and no serious trace of any run in these residuals along the range. (The mean for the two halves are 0·158λ and 0·162λ.) It has seemed better, therefore, on the whole, to apply a constant correction 0·160λ to the Rowland wave-lengths throughout. The fact must be recognised that this method of correcting the Rowland wave-lengths to the I.A. system removes the difference between the positions of lines produced in the laboratory and the Sun, and thereby to a great extent loses the advantage of comparing solar and stellar lines. But until a detailed examination of the differences of wave-length between the solar and terrestrial spectra has been made, this loss would appear to be unavoidable.

The velocities obtained by the various reductions were as follows. They are expressed in km. per sec.

Plate. Reduction.	334	336	337	338	341	Mean Velocity. Km. per Sec.
α=0·25, 12 lines	−7·66	−7·35	−8·21	−7·37	−9·25	−7·97±0·24
α=1·0, 12 lines	−7·79	−7·51	−8·33	−7·58	−9·41	−8·12±0·23
α=1·0, 3 lines	−7·46	−6·87	−7·31	−6·56	−8·62	−7·36±0·23

If Kayser's suggestion of a correction of 0·19λ to Rowland's wave-lengths were adopted, these mean velocities would be reduced to −5·85, −6·00, and −5·24 km./sec. respectively. There seems, however, no justification for adopting his suggestion, in view of the size of the residuals between the two systems in the range of spectrum under consideration, and it is only mentioned here in order that it may be pointed out that there is serious risk of error in its adoption for work of such delicacy as line-of-sight velocity determinations.

It will be noticed that, as was explained in the previous section, the three-line reduction differs markedly in its results from the other two, giving too large a value to the wave-length of the majority of the star lines. The three reductions give nearly the

* *M.N., R.A.S.*, vol. lxiii. p. 299.

† *Astrophysical Journal*, vol. xxxii. p. 217. We may note here two apparent misprints in the table on p. 218. The line 4254·334 should be attributed to chromium and not to calcium, while the line 4271·838 should be 4273·838.

same internal concordance among the separate plates. In the table below are given the Rowland wave-lengths of the star lines used, also the mean discrepancy between the velocity derived from each line and the velocity derived from the mean of the whole plate.

Rowland Wave-length and Element.	Velocity from Plate − Velocity from Line.		
	$\alpha=0{\cdot}25$, 12 Lines.	$\alpha=1{\cdot}0$, 12 Lines.	$\alpha=1{\cdot}0$, 3 Lines.
4203·730 Cr	$-0{\cdot}68\pm0{\cdot}34$	$-0{\cdot}51\pm0{\cdot}45$	$-0{\cdot}33\pm0{\cdot}37$
06·862 Fe	$-0{\cdot}18\pm0{\cdot}34$	$-0{\cdot}02\pm0{\cdot}30$	$-0{\cdot}03\pm0{\cdot}31$
10·528 {·494 Fe, ·561}	$-1{\cdot}37\pm0{\cdot}45$	$-1{\cdot}34\pm0{\cdot}44$	$-1{\cdot}23\pm0{\cdot}46$
19·548 {·516 Fe, ·580}	$+1{\cdot}46\pm0{\cdot}36$	$+1{\cdot}27\pm0{\cdot}34$	$+1{\cdot}14\pm0{\cdot}41$
22·382 Fe	$-0{\cdot}37\pm0{\cdot}40$	$-0{\cdot}58\pm0{\cdot}42$	$-0{\cdot}63\pm0{\cdot}40$
46·996 Sc	$+0{\cdot}52\pm0{\cdot}23$	$+0{\cdot}36\pm0{\cdot}18$	$+0{\cdot}33\pm0{\cdot}31$
50·287 Fe	$-1{\cdot}46\pm0{\cdot}40$	$-1{\cdot}56\pm0{\cdot}39$	$-1{\cdot}58\pm0{\cdot}39$
50·945 Fe	$-0{\cdot}15\pm0{\cdot}62$	$-0{\cdot}18\pm0{\cdot}59$	$-0{\cdot}18\pm0{\cdot}53$
54·505 Cr	$+2{\cdot}93\pm0{\cdot}15$	$+2{\cdot}90\pm0{\cdot}16$	$+2{\cdot}88\pm0{\cdot}18$
94·252 {·204 Ti, ·301 Fe}	$-0{\cdot}98\pm0{\cdot}38$	$-0{\cdot}66\pm0{\cdot}36$	$-0{\cdot}60\pm0{\cdot}44$

The probable errors of the discrepancies of the separate lines from the mean of the plate as given in this table show no real difference between the various reductions.

§ 12. *Final Conclusions.*—The results of the above paper may be briefly summarised as follows. A method is developed by which more than three standard lines can be made use of in reducing a spectrogram with the Hartmann formula. The calculations involved are simple and short; and in the case where α is given any other value than unity, the constants of the curve of reduction are obtained much more rapidly by this method than by the ordinary Hartmann method. There are certain advantages in using more than three standard lines when possible. A comparison has been made for a group of plates reduced in various ways; and it is clear that the new method is not inferior to the old, though the actual figures do not show a marked difference in favour of either way. It should be remembered that this method will rarely be worth applying to a single plate, though it may be used with effect for a large number of plates taken with the same spectrograph and covering the same range of spectrum. It should be particularly useful in work on radial velocities with objective prisms and single-line light-filters.

It remains to express my indebtedness to Professor Newall for the kind and ready way in which he has put his experience and observations at my disposal in the above work.

Cambridge:
1911 *June.*

On the Detection of the Element Dysprosium in the Solar Chromosphere. By Alexander D. Ross, M.A., D.Sc., F.R.S.E., Lecturer on Natural Philosophy in the University of Glasgow.

In 1886 Lecoq de Boisbaudran * announced that the substance hitherto spoken of as the element holmium was in all probability a mixture of two elements which he provisionally designated as Z_γ and holmium proper. The former constituent was defined by five nebulous lines or bands in its absorption spectrum, having wave-lengths of about 5835, 5750, 5700, 5269, and 5259. As Boisbaudran's investigations were carried out on a few decigrammes of material he was unable to isolate the new element, and thus to conclusively demonstrate its existence. After his death no further work was done with regard to it until about 1900, when Urbain, in the first of a series of papers † dealing with his separation of the rare elements, gave an account of certain observations which made its real existence highly probable. The same year Demarçay published his researches on the Terbium group of earths,‡ and stated that he had been able to map several lines in the spectrum of a new element which he provisionally denoted by Δ^3. The separation of almost pure dysprosium (which is the same as Z_γ and Δ^3) was effected by Urbain a few years later,§ and the element was found to give a spectrum exceedingly rich in lines. The author was very kindly furnished by Professor Urbain with samples of dysprosium for the purpose of mapping the spark spectrum and of investigating the Zeeman effect in the case of this element. Four samples were supplied, marked (1) Dy 41; (2) Dy-Ho-Y 49; (3) Ethylsulphate 8; and (4) Ethylsulphate Q. Nos. 1 and 3 were practically pure dysprosium oxide, Dy_2O_3, a very pale yellowish powder. No. 2 contained a considerable admixture of neoholmium and a little yttrium, while No. 4 was fairly pure dysprosium, the chief impurity being terbium. The spark spectrum of each sample was photographed. The negatives from samples 1 and 3 were fully measured for the dysprosium lines, while Nos. 2 and 4 were only investigated for the purpose of sorting out the lines of terbium, neoholmium, yttrium, and erbium, which were present as traces in the purest samples of the dysprosium. Gadolinium was present in such a minute trace as to be recognised only with difficulty, while the other members of the yttrium family with higher atomic weight were either still less noticeable or entirely absent. Accordingly, in mapping the spectrum only four rare elements (viz. Tb, Nh, Y, and Er) were present in such quantity in the dysprosium as to require particular care in separating out lines due to these impurities.

* Lecoq de Boisbaudran, *Comptes Rendus*, cii. p. 153 (1886).
† G. Urbain, *Ann. Chim. et Phys.*, (a) xix. p. 184 (1900).
‡ E. Demarçay, *Comptes Rendus*, cxxxi. p. 387 (1900).
§ G. Urbain, *Comptes Rendus*, cxlii. p. 785 (1906).

The spectroscope employed was a 21-foot radius Rowland grating, having an area of 6 × 2 ins., ruled with 20,000 lines to the inch. The spark was produced by a powerful induction coil, worked by a motor mercury interrupter, and having a battery of Leyden jars connected up in parallel with the sparking apparatus to reduce the electromotive force and increase the quantity of electricity in the discharge, and at the same time to give the discharge its requisite oscillatory character. A self-inductance was introduced into the spark circuit for the twofold purpose of removing the air lines and of rendering the metallic lines as sharp and narrow as possible. The spark spectra of dysprosium and of iron were photographed on the same plates, the two spectra overlapping to a slight extent. Taking the iron lines as a standard, the wave-lengths of the dysprosium lines were determined by interpolation from measurements made by a special form of comparator microscope built to the author's requirements by Zeiss of Jena. Measurements of all lines have been made in the first and second orders, and also, in the case of the more important lines, in the third and fourth orders. Exner and Haschek's* values were taken for the wave-lengths of the iron spark lines; and as a large number of comparison lines have been employed, this should give a sufficiently accurate standard. The differentiation of the overlapping orders has been carried out by an entirely novel method devised by the author, and based on a study of the relative intensities of the components into which the spectrum lines were resolved when the spark was placed in an intense magnetic field.† The full tables of the wave-lengths of several hundred dysprosium lines will be published shortly.

While this work was in progress it was decided to compare the spectrum with that of the Sun in order to see whether any trace of dysprosium could be found in that body. As the spark spectra of the elements are, as a rule, more closely allied to the chromospheric spectrum than the ordinary solar spectrum, it was thought that success would most probably be attained by comparing the dysprosium spark lines with the flash spectra which have been obtained at recent eclipses. The spectra finally employed were those obtained by Mr. Dyson at Ovar and Sumatra during the eclipses of 1900 May 28 and 1901 May 17.‡

The portion of the dysprosium spectrum which has been investigated by the author is the part from wave-length 2400 to 4650. Unfortunately the eclipse photographs did not extend beyond 3300, so that only a limited portion of the two spectra overlapped. Seven lines in this region have been recognised with certainty as due to dysprosium, and probably an eighth line is also present. The following table gives the wave-lengths and intensities of these chromospheric lines, with the corresponding

* F. Exner and E. Haschek, *Wellenlängen-Tabellen der Funkenspectren der Elemente*, Leipzig and Vienna (1904).

† A. D. Ross, *Proc. Roy. Soc. Edin.*, xxx. p. 448 (1910).

‡ F. W. Dyson, *Phil. Trans. Roy. Soc.*, Ser. A, ccvi. p. 403 (1906).

values of the dysprosium lines with which they have been identified :—

The Solar Chromosphere.			Dysprosium.			
			Spark. Ross.		Arc. Eberhard.	
Dyson.		Evershed.				
λ.	Ints.	λ.	λ.	Ints.	λ.	Ints.
3394·71	1	3394·60	3494·61	8	3494·65	6
3531·85	1	3531·83	3531·83	30	3531·86	10
3535·06	1	...	3535·08	10	3535·11	7
3542·58	0	...	3542·44	8	3542·47	5
3550·41	1	...	3550·31	10	3550·36	8
4000·51	1	...	4000·58	30	4000·59	10
4295·20	1	...	4295·14	15	4295·18	8

In two cases where measurements of the chromospheric lines have been made by Evershed, these have been added for the sake of comparison; and in the case of the dysprosium lines, the wave-lengths and intensities both in the spark and in the arc * are set forth in the table. A line at 3645·48 in the chromospheric spectrum has been identified by Dyson with the scandium line at 3645·47. It probably represents both the scandium line and the strong dysprosium line at 3645·50, the two appearing as one in consequence of their very small difference in wave-length.

The eight dysprosium lines referred to above include the chief spark lines, and, in particular, they comprise all the important lines which are decidedly enhanced in passing from the arc to the spark. Dysprosium, accordingly, follows the general rule that the chromospheric lines are mainly enhanced lines, or at least strong spark lines.

In conclusion, the author desires to take this opportunity of thanking Mr. Dyson for the helpful interest which he took in the investigations.

* G. Eberhard, *Publik. des Astrophysikalischen Observatoriums zu Potsdam*, xx. (c), Nr. 60 (1909).

University of Glasgow:
1911 *June* 5.

Spectroscopic Observations of the Sun's Rotation made at the Royal Observatory, Edinburgh, 1908–10. By J. Storey and R. Wilson, B.Sc. (Plate 15.)

(*Communicated by the Astronomer Royal for Scotland.*)

In vol. xli., part i., of the *Proceedings of the Royal Society of Edinburgh*, Dr. Halm gave an account of the observations made by him during the years 1901–3 to determine the period of rotation of the Sun by the spectroscopic method through the displacement of the solar lines.

He continued these observations until his departure for the Cape Observatory in 1907.

A discussion of all these was published by him in *Astronomische Nachrichten*, vol. clxxiii. p. 287, where he advocated a periodic variation in the solar rotation as the result of a comparison of his values with those derived by Dunér at Upsala, who had observed the same lines.

In view of this, it seemed desirable that further observations should be made.

Halm's observations were visual, and he deduced his results from measurements on four lines, namely, two solar and two telluric in the red region of the spectrum. It was conceivable that a greater increase of accuracy might be attained by the adoption of photography, this allowing a larger number of lines to be included, whilst the number of settings of the instrument might be increased.

In the latter part of 1907 some experimental photographs were taken with the heliometer used by Halm. From the measures of these, however, it could not be said that there was any increase of accuracy when only the same four lines were taken; and since the image of the Sun given by the heliometer was only about 0·6 inch in diameter, it was evidently desirable to have a larger image.

Accordingly, the then Astronomer Royal for Scotland (Professor Dyson) ordered from Sir Howard Grubb a long focus object-glass, giving an image of the Sun of about 4·2 inches, and collimator and camera lenses to match. These were delivered during the summer of 1908, and the following is an account of the equipment with which the spectra photographs discussed in this paper have been taken.

The object-glass of 9 inches aperture and 37·7 feet focal length is fixed in an aperture in the northern wall of the optical room, the Sun's light being reflected on to it by a Foucault heliostat exactly as in the apparatus arranged by Dr. Halm.

The grating is a plane one by Rowland, and has 14,438 lines to the inch; it is 5 inches long and $3\frac{1}{2}$ inches broad, but about $\frac{1}{2}$ inch at one end was not used on account of an error in the ruling. The spectrum in the third order has been used throughout, the linear scale being one Angström unit = 1·30 mm.

EQUIPMENT FOR SPECTROSCOPIC DETERMINATION OF SOLAR ROTATION, ROYAL OBSERVATORY, EDINBURGH.

The separation of the spectra in the red of the third order from the violet of the fourth is obtained—on the plan adopted by Professor Newall—by placing a prism of small angle (10°) between the grating and the lens of the camera. This necessitates a tilt of the camera of about 7° to the horizontal.

The collimator and camera lenses, each of 4 inches aperture and 16 feet focal length, are mounted in upright iron frames, made to slide in grooves, the bases of which are bolted to a table, which also supports the grating and the above-mentioned prism. A wooden box encloses these lenses and the grating, and serves to prevent stray light from entering the spectroscope. For economy, the tubes of the collimator and camera telescopes are constructed of cardboard.

The slit is mounted on a metal tube having a rack and pinion adjustment for focussing the collimator. This tube is firmly fixed to a cast-iron table, the top of which can be moved horizontally by a screw in the direction of the length of the collimator. In order to have the slit in the focal plane of the long focus object-glass, an iron rod connects the above-mentioned table-top and the collimator lens, so that by turning the screw the collimator can be moved as a whole in the direction of its length into the required position.

The camera box, with rack and pinion for focussing, takes a plate $6\frac{1}{2} \times 4\frac{3}{4}$ inches, and is so graduated that 18 photographs of spectra about 4 inches long can be obtained on each plate.

To obtain the position angle of any desired point on the Sun's limb, the following arrangement is mounted in front of the slit (fig. 1).

A B C is a brass position circle graduated in degrees from 0 to 360, fixed so that its plane is vertical, and perpendicular to the axis of the collimator. D is a small circular aperture of 6·5 mm. diameter, at the centre of A B C, and directly in front of the slit.

L M N and P Q R are two circular plates rigidly fixed to a bar, with their centres on a diameter of A B C, and capable of rotation in the plane A B C about the centre of D. The circular part *c d e*, in which is the aperture D, is rigidly mounted in front of the slit, and does not rotate when the above-mentioned bar is rotated. X is a pointer, and Y the clamp by which the above circles can be put into any position in the plane A B C.

By means of the slow motions of the heliostat, which are worked by rods carried beneath the floor, and operated by the observer from his position in front of the slit, the Sun's image can be brought centrally with L M N, an exposure made, and immediately afterwards another exposure is made in the position P Q R, the limb being a tangent to the small circle D at the points *a* and *b* respectively. Thus we obtain photographs of diametrically opposite points on the Sun at a distance inside the limb in each case equal to the radius of D. The zero of the position circle is found by placing a level across the centres S

and T, when the reading of the circle should be 90° and 270°; this can be adjusted correctly by means of foot screws.

The photographs were taken on Wratten Panchromatic plates, and the exposures given varied from 3 to 4 minutes according to the time of year.

The measuring machine by Hilger is such that one turn of the head, which is 118 mm. in diameter, carries forward the microscope 1 mm. It is divided to hundredths, and read by estimation to thousandths of a mm.

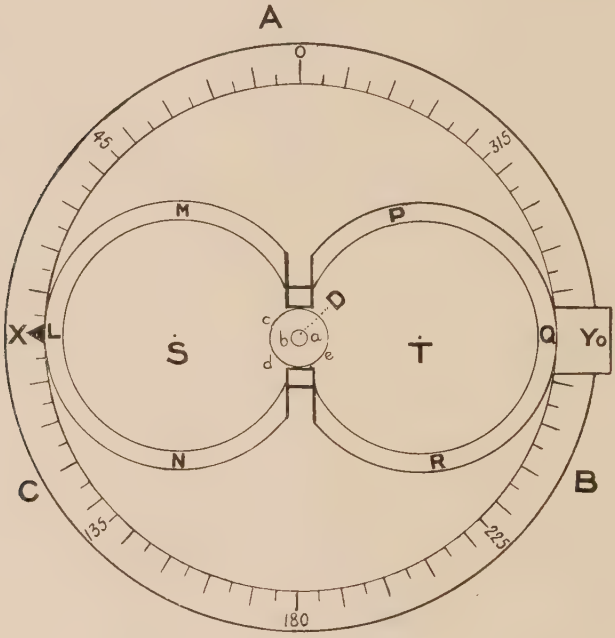


Fig. 1.

In all, twenty different lines, embracing those used by Halm and Dunér, and consisting of nine solar and eleven telluric lines, were measured, the wave-lengths of which, according to Rowland, are given in Table I. Two settings of the micrometer were made on each line, and the mean of the two was adopted. For each pair of exposures the difference of micrometer-reading between the corresponding lines was taken. The mean of these differences for the solar lines was then calculated, and the telluric lines were similarly dealt with. The difference of these two means gives a quantity equal to twice the observed velocity for the particular heliographic latitude, in terms of revolutions of the screw. The value of a revolution was determined from the photographs themselves, in that each gave from the first to the last telluric line a

run of about 45 revolutions, which, divided into the known difference of wave-lengths, gave a very accurate result.

Since the point which gave the spectrum was situated at a distance inside the limb equal to the radius of D, that is, 58″, corrections were applied to the observed velocities in order to reduce them to what they would have been if observed on the limb; and the position angles of the Sun's limb were reduced to true latitude, the various tables as given by Dunér being generally used.

The plates cover the period from 1908 November 30 to 1910 March 17, and the resulting velocities in kilometres per second are given in Table II., where they are grouped for every 5° of heliographic latitude, as represented by ϕ, from 0° to 75°.

The means of these groups are given in Table III., where ϕ represents the mean latitude and ξ the daily angular velocity, the corresponding period in days being given in the last column.

These mean values were then plotted out, and the curve represented in fig. 2, drawn among them.

The velocities taken from this curve for each 5° of latitude are given in Table IV., and denoted by Ed_2. For purposes of comparison the differences of these velocities are also given.

(1) Those derived by Adams at Mount Wilson from photographic observations in the more refrangible region of the spectrum between λ 4196 and λ 4295, published in November 1907, and March 1909. (Contributions from the Mount Wilson Solar Observatory, Nos. 20 and 33.)

(2) Those derived by Halm, denoted by Ed_1, taken from his paper in *Ast. Nach.*, vol. 173, the values used here being obtained from the curve drawn through his mean results for the years 1901–6.

(3) Those derived by Dunér, as given by him in *Ast. Nach.*, vol. 167, the mean of observations taken in 1887–89 and 1899–1901 being used.

A glance at this table shows, in regard to Adams' results, that, with the single exception of the 1906–7 observations at latitude 75°, the Ed_2 observations give a larger rotational value, the maximum difference being at about 40° in 1906–7, and rather later, or about 50°, for the 1908 observations. In each case the curve of the difference is fairly smooth.

With regard to the results obtained by Halm, Ed_1, the comparison shows that the Ed_2 observations give consistently larger values to about 40° of latitude, and smaller for the other portion, the greatest difference in each case amounting to about ·03 km.

For Dunér's values, which are given by him for each 15° of latitude, the Ed_2 observations are in agreement to 30°, and consistently larger for the other three latitudes.

Adams, in the discussion of his 1908 observations, has drawn attention to the agreement of his two sets of observations in middle latitudes, and their discordance in higher latitudes, which might give colour to a variation in the rotation for the latter.

The comparison of the two sets of Edinburgh results as given in column 5 of Table IV. cannot be said to indicate a distinct change in any particular range of latitudes.

With regard to the comparison of Adams' results for 1906 with

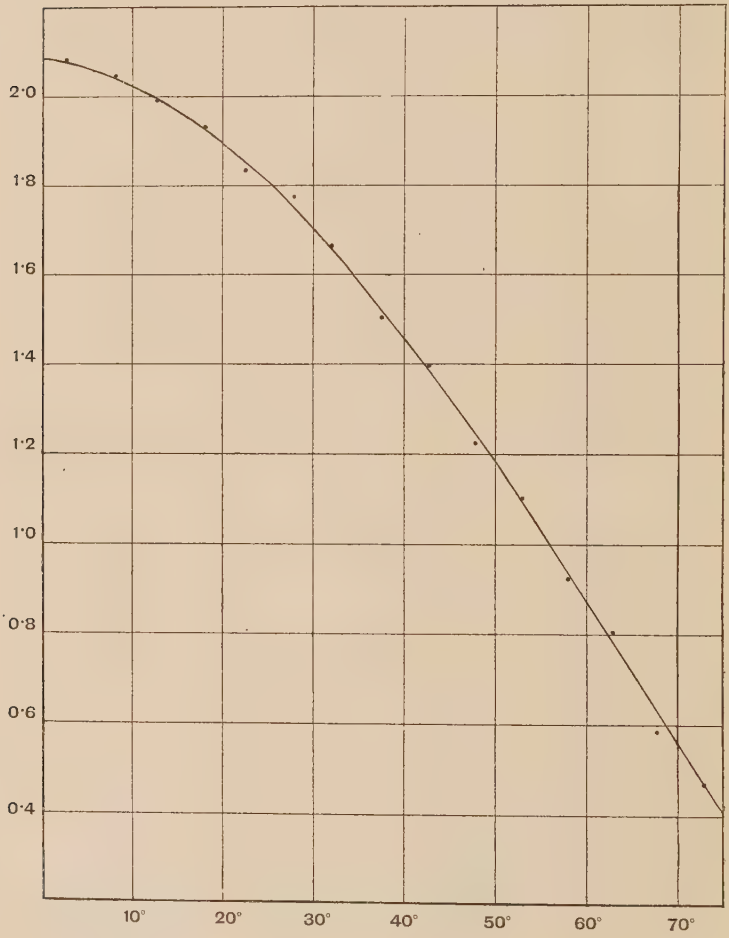


FIG. 2.

our observations, column 3 would tend to show that if any variation took place it was in the middle latitudes.

The differences of Adams' 1908 and our results, as given in column 4, are much more uniform, but still are larger in the middle latitudes.

Table V. gives the results of Table IV. expressed in daily

angular motion, with the addition of the comparison with the rotational values derived from the discussion of the Greenwich sun-spots, 1879–1901, by Mr. and Mrs. Maunder (*Monthly Notices*, vol. lxv.). There is a very good agreement as far as 60° of latitude and somewhat greater discordances above. It is to be noted, however, that the sun-spot results above 35° have been obtained by calculation only.

An investigation was made of the results to see whether they gave any indication of a change with the seasons. No sign of any annual term was found.

Since the angle of inclination of the slit with the limb of the Sun varied from point to point on the limb, the observations of each group in Table II. were arranged with respect to this angle. This examination did not show that the results are in any way affected by the variation in the angle of inclination.

Taking the differences of Tables IV. and V. generally, it would certainly be difficult to assign a periodic variation in the rotation, and we conclude that, whatever may be the cause of these systematic differences, the observations to hand give as yet no definite answer in that direction.

With regard to the probable error of the observations, we find from a group given in Table I. that it is ±·060 km. for a single observation, which is approximately constant for all latitudes, and from this the probable error of a group to be ±·011 km. These are rather larger than one would have expected from the way in which the observations have been made, but may be considered fairly satisfactory in comparison with Halm's ±·070 km. and ±·018 km. respectively, though they are much larger than Adams' ±·015, ±·004 (1906–7), and ±·009, ±·002 (1908), but in these cases 14 and 16 solar lines were used respectively, which doubtless would give greater accuracy than our 9 solar lines.

In addition, an investigation was made to determine whether the comparison of the readings of the individual solar lines would give any systematic differences in the rotation. In no case was there anything found other than could be explained by the accidental error of observation.

The systematic differences between these and other determinations are small, absolutely, being of the order of ·001 mm. upon the micrometer, but while they remain so far in excess of their probable errors, the problem must be felt to stand in an unsatisfactory position.

In order to reconcile the determinations Ed_2 and Adams, it would be necessary to hold that the setting of the Sun's disc upon the hole D, as described above, was systematically closer to the limb than has been estimated. Precautions should be directed to this point, and the observations are to be resumed upon a plan that will remove the difficulty.

TABLE I.

Wave-length.	Origin.	Intensity.	Wave-length.	Origin.	Intensity.
6280·598	A(o)	2	6302·709	Fe	5
6280·833	Fe	3	6302·975	A(o)	2
6281·387	A(o)	1	6310·101	A(o)	2
6290·427	A(o)	2	6310·848	A(o)	1
6291·184	Fe	4	6311·722	Fe	1
6292·373	A(o)	2	6314·449	A(o)	0
6296·170	A(o)	3	6314·876	Ni	4
6298·007	Fe	5	6315·189	A(o)	0
6301·718	Fe	7	6316·028	Fe	1
6302·209	A(o)	2	6318·239	Fe	6

TABLE II.

0°–5°.				5°–10°.				10°–15°.			
Date.		ϕ.	Velocity. km.	Date.		ϕ.	Velocity. km.	Date.		ϕ.	Velocity. km.
1908 Nov.	30	4·1	2·123	1908 Nov.	30	7·0	2·046	1908 Dec.	6	12·5	2·146
Dec.	9	4·9	2·183	Dec.	9	6·7	1·994	1909 Feb.	19	13·6	2·006
1909 Feb.	13	4·9	1·989	1909 Jan.	29	6·0	2·175		19	14·4	2·068
	19	2·3	2·225	Feb.	13	7·5	2·071		20	14·6	1·978
	19	3·3	2·218		19	9·9	2·151		21	13·3	2·099
	20	2·7	2·258		19	9·0	1·996	Mar.	27	14·8	2·097
	21	2·0	2·117		20	9·4	1·991	Apr.	6	14·5	1·953
Mar.	27	4·0	2·192		21	10·0	2·079	May	5	12·3	2·070
Apr.	6	3·2	1·973	Mar.	17	5·5	2·093		7	11·7	2·048
May	5	1·7	1·996		27	9·8	1·909		15	14·5	1·837
	7	2·3	2·082	Apr.	6	8·4	2·097	June	15	11·6	2·010
	24	0·9	2·153	May	5	9·1	2·004		30	11·5	1·737
June	15	1·5	2·127		6	5·5	2·097	July	1	13·5	2·114
	30	1·2	2·009		6	7·3	1·994	Aug.	6	10·5	2·064
July	1	4·5	1·957		7	8·7	1·944		6	13·0	1·997
	1	4·3	2·002		24	9·8	1·968		6	14·9	1·798
Aug.	6	2·4	2·060	June	15	9·0	1·950		8	13·5	1·856
	6	4·5	2·072		30	9·1	2·092		8	13·4	1·751
	8	3·2	2·067	Aug.	6	5·4	2·087		9	12·3	1·856
	9	2·1	2·150		8	6·5	2·009		9	12·6	1·931
	9	2·3	2·046		9	7·5	1·990		9	12·2	1·948
	9	2·0	2·034		9	6·3	2·125	Sept.	1	14·4	1·879
Sept.	1	2·8	2·118	Sept.	1	6·9	2·103		13	10·5	2·143
	15	0·6	1·935		14	6·6	2·099		15	12·4	2·029
	17	1·6	1·975		15	9·1	1·937		17	12·4	2·028
Oct.	4	0·6	2·056		17	8·1	2·097	Oct.	15	12·0	1·928
	15	0·1	1·979	Oct.	4	9·6	1·925		15	10·7	2·143
Nov.	15	0·3	2·134	1910 Feb.	14	9·9	1·963	Nov.	15	10·3	2·112
1910 Feb.	14	2·1	1·977		23	6·9	2·213		15	11·3	1·847
	15	1·1	2·183	Mar.	1	8·3	2·151	1910 Feb.	14	12·9	2·052
	23	5·0	2·230						15	11·0	2·053
Mar.	17	3·2	2·014					Mar.	17	14·0	2·095

TABLE II.—*continued.*

15°–20°.				20°–25°.				25°–30°.			
Date.			Velocity. km.	Date.			Velocity. km.	Date.			Velocity. km.
1908 Nov.	30	17·3	2·037	1908 Dec.	6	24·5	1·928	1908 Nov.	30	27·3	1·675
	30	18·1	1·817	1909 Feb.	19	24·8	·857		30	29·0	·727
Dec.	9	17·4	2·071		20	21·4	·920	Dec.	9	28·1	·846
1909 Jan.	19	19·3	1·902		21	24·6	·753	1909 Jan.	29	28·7	·737
	29	17·4	2·038		21	21·3	·876	Feb.	19	26·1	·797
Feb.	13	19·6	1·915	Apr.	14	21·8	·904		20	25·7	·807
	19	20·0	1·927	May	5	22·7	·875		21	30·0	·824
Mar.	17	16·5	1·991		7	23·3	·799	Mar.	17	27·6	·837
	27	19·6	2·017		15	23·9	·811		27	25·6	·833
Apr.	6	19·2	1·953	June	15	22·0	·771	Apr.	6	29·9	·740
May	5	19·7	·940		30	21·5	·619		6	25·2	·958
	6	16·1	·917	Aug.	6	20·8	·945	May	6	27·2	·728
	6	18·5	·974		8	23·7	·773		6	28·9	·825
	7	19·3	·935		9	23·1	·748	June	30	29·6	·858
June	15	16·4	·987		9	22·6	·759	July	1	27·6	·814
	30	19·3	·883	Sept.	1	22·8	·700		1	27·0	·749
July	1	15·8	·906		13	21·1	·883		1	29·8	·838
Aug.	6	15·8	·777		15	22·8	1·809	Aug.	6	26·6	·862
	7	16·7	·939		17	22·7	2·029		6	25·1	·614
	8	16·8	·918	Oct.	4	20·2	1·708		6	26·2	·894
	9	17·9	·927		4	22·3	·832		7	27 2	·718
	9	17·7	·775		15	22·8	·916		8	27·3	·848
Sept.	14	17·3	·803		15	22·6	·897		9	28·1	·714
	15	19·6	·891	Nov.	15	22·3	·800		9	28·0	·739
	17	19·4	·816	1910 Feb.	14	20·8	·952	Sept.	14	28·1	·682
Oct.	8	17·3	1·838		15	22·1	1·781		17	29·9	·656
1910 Feb.	23	17·9	2·039					Oct.	8	28·1	·779
	23	16·3	2·018						20	28·8	·764
Mar.	1	20·0	1·849					Nov.	8	26·7	·744
	15	16·7	1·987					1910 Feb.	23	28·8	·686
	17	19·5	2·070					Mar.	1	28·0	·775
									15	27·8	1·816

TABLE II.—*continued.*

30°–35°.				35°–40°.				40°–45°.			
Date.		ϕ.	Velocity. km.	Date.		ϕ.	Velocity. km.	Date.		ϕ.	Velocity. km.
1908 Dec.	6	33·5	1·758	1908 Nov.	30	38·7	1·467	1908 Nov.	30	40·4	1·577
1909 Jan.	19	30·8	·732	Dec.	9	37·1	·310	Dec.	6	44·7	·408
Feb.	13	31·6	·656	1909 Jan.	29	40·0	·519	1909 Jan.	19	42·0	·451
	19	30·1	·770	Feb.	19	35·9	·695	Feb.	13	42·5	·445
	20	32·3	·701		19	37·5	·624		19	42·7	·344
	21	30·5	·759		20	36·5	·616		20	41·0	·590
Mar.	5	32·3	·738		21	36·2	·539		20	43·1	·395
	27	31·8	·730	Mar.	17	38·5	·526		21	41·2	·523
Apr.	6	30·1	·618	May	6	37·3	·521	Mar.	5	43·6	·364
	15	32·3	·683		15	35·2	·396		17	42·2	·369
May	5	33·3	·578	July	1	37·9	·416	Apr.	6	40·5	·536
	5	30·1	·716		1	36·3	·313		6	41·0	·439
	7	33·8	·655		1	36·1	·492		15	42·6	·609
June	15	33·4	·613		13	35·4	·638	May	5	40·4	·502
	30	31·9	·736	Aug.	6	36·7	·688		6	40·3	·428
Aug.	6	31·3	·615		6	35·5	·283		7	40·7	·392
	9	33·3	·445		6	36·5	·521		7	44·4	·219
	9	33·2	·517		7	39·2	·368		8	45·0	·272
Sept.	13	31·9	·742		8	37·7	·637	June	15	43·9	·405
	15	30·3	·770		8	35·4	·540		30	44·0	·254
	15	34·0	·567		9	38·4	·572	Aug.	6	41·6	·378
	17	33·3	·727		9	38·2	·488		7	44·5	·339
Oct.	4	30·8	·610	Sept.	14	38·6	·472		9	43·7	·223
	4	32·9	·603		16	37·6	·482		9	43·4	·270
	15	33·6	·624	Oct.	4	37·5	·514	Sept.	13	42·3	·398
Nov.	15	33·5	·454		8	39·1	·354		15	41·7	·489
1910 Feb.	9	30·7	·623		20	39·7	·595		17	40·5	·431
	14	31·7	·717	Nov.	8	37·9	·445		17	44·1	·329
	15	33·1	·736	1910 Mar.	1	38·7	·593	Oct.	4	41·9	·325
Mar.	1	30·9	·629		15	38·5	1·478		4	43·5	·294
	17	30·3	1·736						15	44·7	·290
								1910 Feb.	9	41·9	·342
									14	43·2	·448
									15	44·3	·347
								Mar.	1	42·1	1·456

TABLE II.—*continued.*

45°–50°.				50°–55°.				55°–60°.			
Date.		ϕ.	Velocity. km.	Date.		ϕ.	Velocity. km.	Date.		ϕ.	Velocity. km.
1908 Dec.	9	48·2	1·049	1908 Nov.	30	52·9	0·940	1908 Dec.	6	55·3	0·990
Feb.	19	47·0	·281		30	52·3	1·073	1909 Feb.	19	58·1	·834
	19	48·8	·305	1909 Feb.	13	53·4	·125		19	60·0	0·897
	20	47·9	·258		19	53·2	·055		21	58·9	1·079
	21	47·0	·335		20	52·3	·237		21	58·7	0·915
	21	48·3	·147		21	52·7	·134	May	2	56·6	0·966
Mar.	3	49·1	·369	Mar.	5	54·7	·152		6	59·2	0·906
	17	49·6	·191		17	53·1	·041		8	56·2	1·047
May	2	46·1	·199	Apr.	6	51·3	·190	July	1	58·5	1·030
	6	48·5	·107		6	52·2	·113		1	57·9	0·876
	8	45·6	·334		15	52·7	·241		1	57·4	0·987
July	1	48·3	·345	May	5	50·7	·156	Aug.	6	57·0	1·029
	1	47·5	·280		6	50·8	·148		6	55·9	0·875
	1	45·9	·086		7	51·3	1·081		7	59·4	0·742
	13	45·7	·266		8	54·4	0·987		8	58·4	1·154
Aug.	6	46·9	·353	June	15	54·2	1·028		8	55·5	0·802
	6	45·6	·134	July	13	52·9	·099		9	58·9	·874
	6	46·8	·282	Aug.	6	51·9	·154		9	59·9	·789
	7	49·5	·064		7	54·7	1·042	Sept.	13	59·4	·703
	8	48·0	·269		9	53·8	0·992	Oct.	4	58·6	·975
	8	45·5	·222		9	53·7	0·999	1910 Jan.	21	58·7	0·854
	9	49·0	·329	Sept.	13	52·6	1·139	Feb.	15	55·2	1·041
	9	48·5	·289		15	51·3	·333				
Sept.	14	49·1	·045		17	50·9	1·230				
	16	48·2	·157	Oct.	4	53·8	0·908				
Oct.	4	48·2	·398		4	54·7	1·104				
	8	49·8	·143		20	50·3	·300				
Nov.	15	45·8	·092		31	52·5	·063				
1910 Feb.	23	49·4	·110	1910 Feb.	9	53·0	1·080				
Mar.	1	49·4	·248		14	54·1	0·897				
				Mar.	1	52·7	1·088				

TABLE II.—*continued.*

60°–65°.				65°–70°.				70°–75°.			
Date.		ϕ.	Velocity. km.	Date.		ϕ.	Velocity. km.	Date.		ϕ.	Velocity km.
1908 Nov.	30	64·5	0·766	1908 Dec.	6	66·5	0·652	1908 Nov.	30	74·3	0·472
	30	63·5	·736		9	66·5	·664	1909 Feb.	20	74·9	·482
1909 Feb.	13	64·3	·690	1909 Feb.	19	69·1	·588		20	70·7	·454
	20	63·9	·835	May	2	67·7	·644		21	75·0	·433
	21	63·9	·784		6	69·5	·407		21	70·4	·597
Mar.	17	61·3	·867		6	66·0	·790	Mar.	17	71·9	·540
	17	63·7	·768		15	66·5	·611		17	74·4	·285
Apr.	6	61·8	·893	July	1	68·4	·577	Apr.	6	72·6	·460
	6	64·1	·693		1	68·4	·458		6	74·9	·495
	15	64·0	0·942		1	67·3	·775		15	75·0	·455
May	5	61·8	1·002		1	70·0	·592	May	5	72·1	·614
	6	61·3	0·898	Aug.	6	66·6	·655		6	72·1	·588
	7	62·8	·786		6	67·3	·579		7	73·6	·418
	15	61·2	·846		7	69·2	·399		15	71·6	·508
June	15	64·3	·739		8	68·0	·698	Aug.	7	73·4	·367
July	1	60·1	·895		8	65·4	·473		8	74·3	·215
Aug.	7	64·5	·802		9	69·4	·545		9	70·1	·495
	9	63·6	·794	Sept.	13	68·8	·313		9	72·7	·406
Sept.	13	62·6	·796	Oct.	4	68·5	·473		9	73·8	·347
	15	61·4	·870		4	65·4	·664	Sept.	13	71·8	·375
	17	60·6	·708	1910 Jan.	21	69·8	·648		15	70·6	·626
Oct.	4	63·5	·693	Feb.	14	67·2	·739		17	70·3	·654
	20	60·9	·948		15	66·2	0·573		17	70·5	·350
	21	63·4	·666					Oct.	4	73·4	·467
1910 Feb.	9	64·3	0·668						4	74·4	·399
	23	60·3	1·046						20	70·7	·486
Mar.	1	63·7	0·654					1910 Jan.	18	74·9	·464
	1	60·3	0·861					Mar.	1	74·8	·580
									1	70·9	0·645

TABLE III.

ϕ.	Velocity.	No. of Obs.	ξ.	Period.
°	km.		°	days.
2·6	2·082	32	14·80	24·32
8·0	2·045	30	14·68	24·52
12·7	1·990	32	14·48	24·86
18·0	1·931	31	14·41	24·98
22·5	1·833	26	14·11	25·51
27·7	1·778	32	14·24	25·28
32·0	1·663	31	13·92	25·86
37·5	1·503	30	13·45	26·77
42·6	1·397	35	13·47	26·73
47·8	1·223	30	12·93	27·84
52·7	1·101	31	12·89	27·93
57·9	0·926	22	12·38	29·08
62·7	0·809	28	12·52	28·75
67·7	0·588	23	11·00	32·73
72·8	0·472	29	11·33	31·77

TABLE IV.

Lat.	Ed_2.	Ed_2 – Adams.		$Ed_2 - Ed_1$.	Ed_2 – Dunér.	Ed_2 – Gr.
	v. 1908–10.	Δv. 1906–7.	Δv. 1908.	Δv. 1901–6.	Δv. 87–9, 99–01.	
°	km.	km.	km.	km.	km.	
0	2·086	+ ·009	+ ·026	+ ·033	·00	
5	·065	·009	·022	·029		
10	2·024	·012	·024	·024		
15	1·968	·014	·025	·016	·00	
20	·896	·024	·033	·016		
25	·808	·040	·047	·018		
30	·705	·045	·046	·026	·00	
35	·585	·053	·048	·019		
40	·458	·052	·044	+ ·012		
45	·321	·047	·047	– ·007	+ ·05	
50	·178	·051	·052	·015		
55	1·027	·036	·060	·016		
60	0·875	·024	·055	·017	+ ·06	
65	·719	·008	·042	·017		
70	·561	+ ·008	·028	·026		
75	0·410	– ·019	+ ·014	– ·030	+ ·02	

Table V.

	ξ.	Δξ.	Δξ.	Δξ.	Δξ.	Δξ.
°	°	°	°	°	°	°
0	14·81	+0·06	+0·18	+0·23	0·00	+0·21
5	·72	·06	·16	·21	...	·15
10	·59	·09	·17	·17	...	·08
15	·46	·10	·18	·12	·00	·05
20	·32	·18	·25	·12	...	·04
25	14·15	·31	·37	·14	...	·04
30	13·97	·37	·38	·21	·00	·06
35	·74	·46	·42	·16	...	·05
40	·52	·48	·40	+0·11	...	·05
45	·26	·48	·48	−0·07	+0·50	·03
50	13·01	·56	·58	·16	...	+0·02
55	12·71	·45	·75	·20	...	−0·05
60	·43	·34	·78	·24	·85	·12
65	12·04	·13	·70	·28	...	·31
70	11·64	+0·16	·58	·54	...	·53
75	11·24	−0·53	+0·38	−0·82	+1·57	−0·80

On the Law which governs the Variations of SS Cygni: Preliminary paper. By E. T. Whittaker, Sc.D., F.R.S., Royal Astronomer of Ireland.

In the case of those variable stars which are strictly periodic, a definite cycle of changes is continually repeated at regular intervals of time; and when the cycle has been determined once for all by observations made during a few periods, the brightness at any future instant may be confidently predicted. In the case of "irregular" variables, the law of succession of change is not obvious and the means of prediction are wanting. It is the aim of the present investigation to supply this defect so far as concerns the well-known irregular variable SS Cygni.

The fact that the brightness of any "regular" variable can be expressed mathematically as a sum of sine and cosine terms leads to the obvious conjecture that the brightness of irregular variables may be expressible in a somewhat similar fashion, though evidently the several terms cannot now all have the same period. Adopting this conjecture provisionally, it will be desirable first to set forth the different alternatives which may be imagined, and which must be tested by examining whether their consequences agree with the observed facts.

It might be, for instance, that the variation of SS Cygni resembles, so far as its mathematical expression is concerned, the variation in the height of the sea produced by the tides, which

may be represented by a number of sine and cosine terms having different periods — the "semi-diurnal tide," "diurnal tide," "fortnightly tide," etc. If this should prove to be the case, it would only be necessary to determine these sine and cosine terms, and the changes of brightness of the star in the future could then be predicted.

On the other hand, the variations of SS Cygni might be analogous rather to the variations in the temperature at any terrestrial station: these may be regarded as formed by the superposition of capricious irregularities on certain regular changes whose periods are one day and one year. No sharp line of demarcation can be drawn between this class and the preceding, for it is possible to imagine a continuous series of connecting forms in which capricious irregularities are less and less conspicuous, and regular periodicities more and more dominant. The variation of spottedness on the Sun must be placed somewhere in this sequence: its regular periodicities are those of 11·1 years, 4·79 years, etc., which have been investigated by Dr. Schuster.*

Lastly, the variations of SS Cygni might be conceived to be wholly capricious; a supposition against which the mind naturally revolts, and which will, in fact, presently be shown to be untrue.

In order to decide between these various possibilities, it is necessary to analyse the variations of brightness of SS Cygni in the same way as variations in the displacement of the æther are analysed in a spectroscope. As no instrument has yet been devised for performing the analysis as a whole automatically, recourse must be had to arithmetical processes:† an example of the methods which may be employed is afforded by Dr. Schuster's admirable researches on the periodicity of sun-spots, to which reference has already been made. A "curve of periods"‡ will thus be obtained which will correspond exactly to a spectrum; the only difference being that the variations whose analysis is depicted in a spectrum take place in the æther and have periods of the order of 10^{-15} seconds, whereas the variations whose analysis is depicted in the curve of periods take place in the luminous intensity of a star and have periods of the order of days and years.

When the curve of periods has been obtained, a mere inspection of it is sufficient to settle the main question as to the nature of the star's variation. For the curve of periods may be either a curve of continuous curvature, or it may rise in a discontinuous fashion into a succession of sharp peaks. The former corresponds to the continuous spectrum which is obtained when white light is analysed

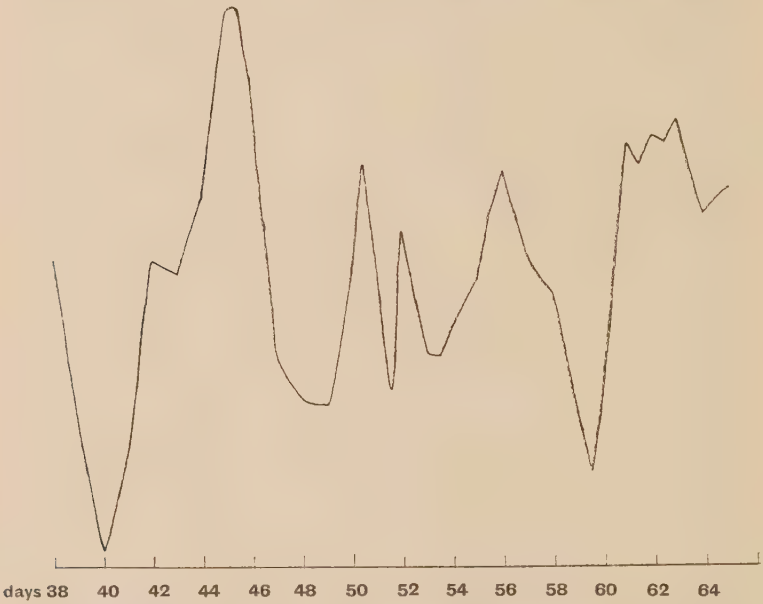
* Schuster, *Trans. Camb. Phil. Soc.*, xviii. (1900), p. 107; *Phil. Trans.*, ccvi. (1906), p. 69: *Proc. Roy. Soc.*, lxxvii. (1906), p. 136; lxxxv. (1911), p. 50.

† Supplemented, of course, by such machines as arithmometers and harmonic analysers.

‡ My reason for not using Schuster's excellent names "periodograph" and "periodogram" is that my curve of periods is not formed in accordance with his definition. I have followed a process which, though less elegant and complete, is more rapid (a great consideration when the undertaking is in any case very laborious), and which leads to the same end.

in a spectroscope, while the latter corresponds to the discontinuous or "line" spectrum which is obtained from the light of a Geissler tube filled with hydrogen. In the former case, it will not be possible to represent the star's variation by a finite number of sine or cosine terms; in the latter case, it will be possible.

Let us now apply these principles to the case of SS Cygni. Working with the observational data published by Col. E. E. Markwick,* Mr. C. L. Brook,† and (in part) Mr. Leon Campbell,‡ the present writer has calculated that part of the curve of periods



Analysis for the detection of Periodic Terms in the Variation of SS Cygni. The peaks indicate the existence of several terms, of which the largest has a period of 45·3 days.

which is comprised between the limits of 38 and 65 days (see the diagram); the ordinate corresponding to each abscissa T being derived from a sequence of observations extending over an interval of time 18 T. It will be seen that the main question at issue is solved immediately by the form of the curve: this consists of a succession of well-defined peaks, the most prominent of which occupies the position corresponding to 45·3 days. It follows that *the brightness of the star SS Cygni may be represented by a sum of sine and cosine terms, the most important of those here indicated having a period of* 45·3 *days.*

* *Jour. Brit. Ast. Assoc.*, xviii. p. 243; xix. p. 201; xx. p. 295.
† *Ibid.*, xxi. p. 255.
‡ *Annals Harvard Coll. Obs.*, lxiv., No. ii.

The breadth of the peaks depends on the extent of the light-curve used in the calculation: in the present work this is such as to include 18 consecutive periods of the terms investigated. If a smaller extent of the light-curve were used, the peaks, being broader, would tend to merge into each other, just as the lines of a spectrum tend to merge into each other when the resolving power of the spectroscope is diminished; while if a greater extent of the light-curve were used, the peaks, being narrower, would (with their attendant "spurious peaks," which in the optical analogue are represented by diffraction-effects) be more clearly separated. As the matter stands, the peaks are so close together as to be affected by each other; and it is therefore intended, in the continuation of this work, to use a greater extent of the light-curve in order to effect a more decided separation.

The influence of the individual sine and cosine terms indicated by the peaks can without much difficulty be traced in the light-curve of the star. As an illustration of this, we may consider the term having the period 45·3 days. The minima and maxima of this term during the years 1908 and 1909 were as follows:—

Date of Minimum of Term.	Julian Day.	Observed Magnitude of Star.	Date of Maximum of Term.	Julian Day.	Observed Magnitude of Star.
1908 Jan. 15	7956·4	11·9	1908 Feb. 4	7976·4	8·7
Feb. 29	8001·7	11·6	Mar. 20	8021·7	11·2
Apr. 15	8047·0	11·0	May 5	8067·0	9·6
May 30	8092·3	11·9	June 19	8112·3	11·2
July 14	8137·6	11·3	Aug. 3	8157·6	11·3
Aug. 28	8182·9	11·3	Sept. 17	8202·9	8·4
Oct. 13	8228·2	10·4	Nov. 2	8248·2	10·4
Nov. 27	8273·5	11·0	Dec. 17	8293·5	8·9
1909 Jan. 11	8318·8	10·8	1909 Jan. 31	8338·8	9·4
Feb. 26	8364·1	11·1	Mar. 18	8384·1	8·7
Apr. 12	8409·4	11·1	May 2	8429·4	8·7
May 27	8454·7	11·2	June 16	8474·7	9·4
July 12	8500·0	11·2	Aug 1	8520·0	8·5
Aug. 26	8545·3	11·7	Sept. 15	8565·3	11·6
Oct. 10	8590·6	11·5	Oct. 30	8610·6	11·4
Nov. 24	8635·9	11·5	Dec. 14	8655·9	9·2
Average observed brightness of the star at the 16 minima of this term		11·3	Average observed brightness of the star at the 16 maxima of this term		9·8

So many other sine and cosine terms contribute to the light-curve that we cannot expect this particular term to be very clearly disentangled from them by inspection; but the above table shows that there is an unmistakable tendency for the star to be faint at

the minima and bright at the maxima of the term: the difference in favour of the maxima is, on the average, $1\frac{1}{2}$ magnitudes, which represents the range of this 45·3-day term.

The figure also indicates the existence of a term having a period of approximately 56 days, whose maxima are at the Julian days 8369·5, 8425·5, etc. The maxima of the star which were observed at the beginning of February 1908, the middle of September 1908, the end of April 1909, and the first half of December 1909 appear to be due largely to the concurrence of maxima of the 45·3-day term and the 56-day term; while in the maxima of 1908 July 23, 1908 November 13, and 1909 October 14 the 56-day term received little or no help from the 45·3-day term; and in the maxima of December 1908, and of early February, March, June, and early August 1909, and the slight rises about 1908 June 20 and 1908 November 2, the 45·3-day term received little or no help from the 56-day term; but until the other terms are taken into account it is not safe to carry these comparisons too far.

Regarding the periodicity of about 64 days, and other periodicities whose existence is indicated by the figure, it is as yet too early to speak definitely.

The question may be asked as to whether the sine and cosine terms, which are thus found to represent the variation of the star, correspond individually to physical causes which produce the variation. Nothing definite can be said on this point, but the probabilities seem to be against the supposition that the several trigonometric terms have independent physical origins; for, in the case of the tides, it is known that the trigonometrical term representing the semi-diurnal tide is produced by no single physical cause, but by the joint action of the Sun (which also contributes the tides of periods six months, etc.) and the Moon (which also contributes the tides of period one fortnight, etc.). There is, however, a great advantage gained by expressing the variation as a mathematical formula; for we thereby acquire the power of prediction, and become able to issue ephemerides of irregular variables in advance. Moreover, there can be little doubt that the determination and comparative study of the mathematical formulæ which represent the fluctuations of different irregular variables—what we may call the Keplerian aspect of the problem—is a necessary preliminary to what we may call the Newtonian aspect, namely, the physical theory.

The computations are now being carried out, and it is hoped in a subsequent paper to complete the curve of periods, calculated from a more extended portion of the light-curve, and to give tables of the sine and cosine terms thus indicated, with an ephemeris based on them.

Dunsink Observatory, Castleknock:
1911 *June* 6.

Measures of the proper motion Star, Sh 190.

By S. W. Burnham, M.A., Assoc. R.A.S.

As a double star, this was first observed by Sir William Herschel in 1791, and subsequently by South and Herschel, who measured the angle and distance in 1823, and entered it as No. 190 of their catalogue of double stars. This is No. 7060 of my *General Catalogue of Double Stars*. The place (1880) is

R.A. $14^h\ 50^m\ 27^s$
Decl. $-20°\ 52'$

It was soon found that these stars, called seventh and eighth magnitude in Sh, had an unusually large proper motion of about 2″ a year, and that there was very little relative change, thus making a system of the 61 Cygni type, with a movement in space exceeded only by the famous pair in Cygnus.

In 1878 I measured four very small stars in the field from A for the purpose of getting later an independent value of the proper motion of the large star. It is practically certain that no one of these faint stars can have any sensible movement. No star of this class anywhere, not attached to and moving with a much brighter star, has been found to have any proper motion whatever.

The recent measures of the several distant stars are as follows :—

A and C.

1911·219	285°·1	26″·74
·241	284·4	26·84
1911·23	284·7	26·79

A and D.

1911·184	11°·82	101″·13
·219	11·12	101·53
·222	11·67	101·44
1911·21	11·54	101·37

A and E.

1911·184	191°·23	45″·35
·219	191·68	45·38
·222	192·45	45·64
·241	192·07	45·34
1911·21	191·86	45·43

A and F.

1911·184	324°·55	188″·49
·219	324·55	188·87
·222	324·75	188·67
1911·21	324·62	188·68

The change in the position-angle of C since 1878 amounts to more than 113°. It is now very near B, and consequently very difficult to see and measure.

Comparing these positions with those of 1878, given in Part II., we have for the proper motion of A—

AC	R.A. +1″·037	Decl. −1″·781
AD	1·057	1·735
AE	1·032	1·768
AF	1·056	1·757
	1·045	1·760

Therefore the corrected proper motion of A is

2″·047 in 149°·30.

Some of the later determinations from meridian observations of the proper motion of the principal star are

Bonn	2″·014 in 151°·27
Porter	2·064 ,, 150·47
Boss	2·039 ,, 149·35

For the present relation of the large stars

A and B.

1911·260	296°·14	17″·88
·280	296·75	17·56
·299	296·19	17·53
·302	296·15	17·86
1911·28	296·31	17·71

From the measure by Herschel in 1836, and seven subsequent positions by Hl, Doo, and β on 26 nights, we have for the apparent motion of B—

0″·101 in 326°·67.

Hence the real proper motion of this star is

1″·948 in 149°·45.

It is quite possible that all, or many, of the double stars of this type, of which 61 Cygni is the best known example, when the measures have been continued for a sufficient length of time, will show that the observed path of the companion star is really a curved line; but a long period will be necessary in order to eliminate the unavoidable errors of observation in the most careful work with the micrometer.

Yerkes Observatory:
May 15.

On the Galactic Distribution of Gaseous Nebulæ and of Star Clusters. By Arthur R. Hinks, M.A. (Plate 16.)

§ 1. In the *Monthly Notices* for 1911 May, I have given the result of a preliminary inquiry into the galactic distribution of spiral nebulæ. I now beg leave to present to the Society the results of a similar inquiry into the galactic distribution of the gaseous nebulæ and of star clusters.

It will be understood that this inquiry is in the nature of a reconnaissance. When one tries to sort out the very mixed material available, one finds a number of objects which it is impossible to class. Setting these aside for a moment, one finds that the more satisfactory part of the material gives clear indications of distribution laws, which would be of decided interest if they could be verified. Going back to the objects which have been set aside, one can estimate how important they will be in a more complete investigation; one can see to some extent how to distinguish between the important and the unimportant; and one can take steps to procure at once the further information desired.

§ 2. *The nebulæ in the Greater Cloud of Magellan.*

In my previous paper it was remarked that no spiral nebula is known in the greater cloud of Magellan, and that it would be of great interest to know whether the nebulæ in that cloud are gaseous. Since that paper was written I have obtained important evidence on the matter in a singular way.

By chance I noticed that

Index Catalogue 4274 = N.G.C. 5189.

The former is No. 93 in the list of "Nebulæ discovered at the Harvard College Observatory," *Annals*, vol. lx., No. vi. It was announced in Harvard Circular No. 60 among "objects having peculiar spectra." This circular begins, "A list of stars having peculiar spectra . . . ." And the object is described as "gaseous nebula." Dr. Dreyer seems to have supposed that the object was of the same class as the many truly stellar gaseous nebulæ previously announced from Harvard; and did not notice that its place is identical with that of the nebula N.G.C. 5189, described

as ! B, pl, cE, bM, curved axis, and figured by Sir John Herschel in *Results, Cape of Good Hope*, plate vi. fig. 1.

Working on this clue, I looked up the other "planetary stellar" nebulæ in the Index Catalogue, and was happy to find the following identities :—

Index Cat.	2105 (Harvard List	84) =	N.G.C.	1698 pB, pS, R, Gl bM.
	2111 (,, ,,	85) =		1722 Cl, pF, S, R, 2nd. of 3.
	2114 (,, ,,	86) =		1748 pB, vS, R, 4th. of 4.
	2115 (,, ,,	87) =		1763 vB, vL, viE.
	2116 (,, ,,	88) =		1769 B, L, iR, vsm bM.
	2117 (,, ,,	89) =		1770 Cl + neb. pl, pRi.
	2126 (,, ,,	90) =		1935 pF, S, R, 3rd. of group.
	2127 (,, ,,	91) =		1936 ! pB, S, R, 4th. of group.
	2145 (,, ,,	92) =		2086 B, pS, R, lbM.
Also	4816 (,, ,,	81) =	Nova Sagittarii 1898.	
	4850 (,, ,,	82) =	Nova Aquilæ 1901.	

This was an excellent addition to the very small stock of information previously in hand concerning the nebulæ in the greater cloud ; it lends some probability to the idea that all the principal nebulæ in the cloud may be gaseous.

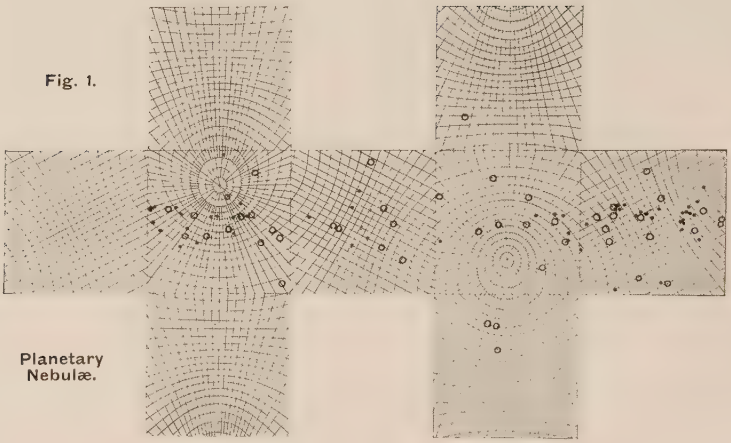
§ 3. *The galactic distribution of "stellar planetary" and planetary nebulæ.*

I use the words "stellar planetary" in the sense that the object has no sensible diameter under a low power, and that its spectrum is known to be gaseous. Copeland discovered a few of these objects. Very nearly all the others were found by Professor Pickering and Mrs. Fleming on Harvard photographs. Omitting those from above, List I. contains the known objects of this class.

List II., of the planetary nebulæ, includes only objects whose spectrum is said to be gaseous. (The authority for the statement in many instances is the list given in Frost's Scheiner, *Astronomical Spectroscopy*, p. 232.) The greater part of these are marked in the N.G.C. with the sign for planetary—O ; I have added a few more to the list where photographs or descriptions seemed to make it necessary to class them as "planetary." Included in the additions are several "nebulous stars," in the Herschelian sense,* that is, sharply defined stars in large gaseous atmospheres. Since a great number of the objects ordinarily classified as planetary nebulæ have a stellar nucleus with a continuous spectrum, it does not seem necessary at present to draw a possibly unreal distinction between them and the nebulous stars.

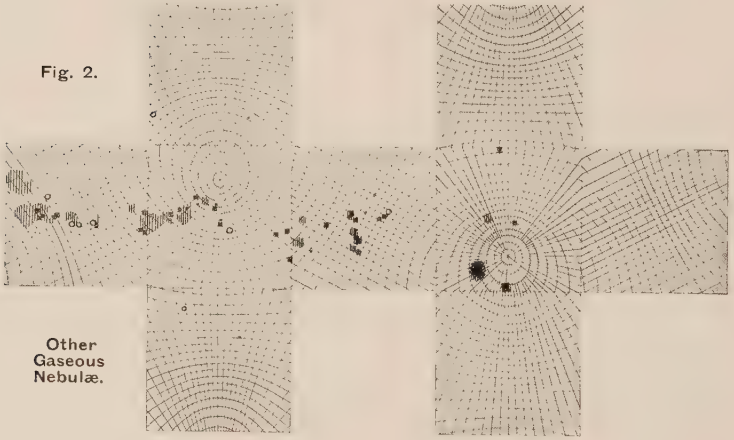
* Professor Solon Bailey has some valuable remarks in *Harvard Annals* vol. lx. page 201, on a recent regrettable abuse of the term "nebulous star." It should not be used to describe any small, hazy, somewhat starlike object.

Fig. 1.



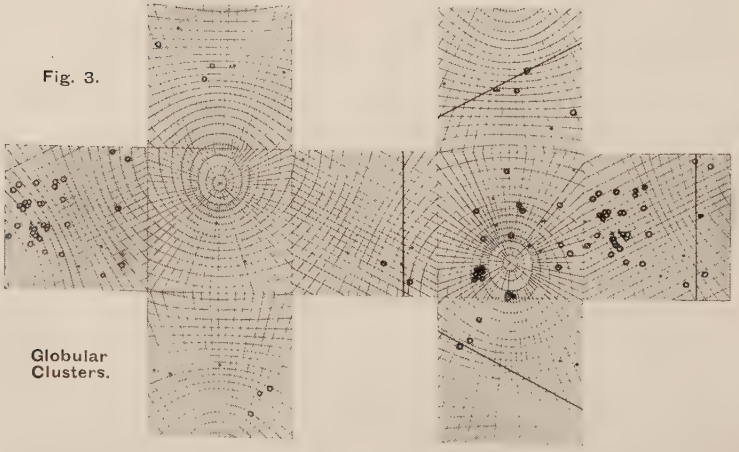
Planetary Nebulæ.

Fig. 2.



Other Gaseous Nebulæ.

Fig. 3.

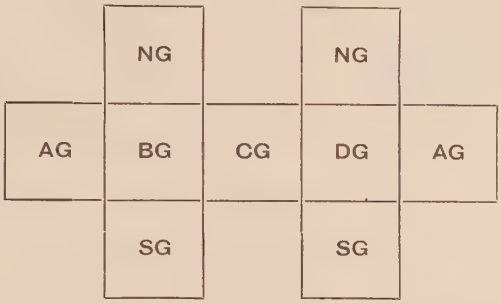


Globular Clusters.

A few objects given as planetary or ? planetary in the N.G.C. are omitted, since the evidence for their gaseous nature is wanting.

The objects in List I. are plotted in Plate 16, fig. 1, as dots; those in List II. as circles.

As before, we will denote the squares of the figure thus,



the galactic poles being in the centres of NG and SG.

It is pretty clear from the figure that—

(1) the planetary and stellar gaseous nebulæ are galactic objects.

I use the word "galaxy" in a broad sense, and do not confine its meaning to the narrow zone of the Milky Way proper.

(2) the stellar are more condensed on the galaxy than the planetary. This is consistent with the view that the stellar planetaries are ordinary planetaries too far away to show a visible disc.

(3) both classes are more numerous, and more condensed upon the galaxy, in AG, DG, than in BG and CG.

(4) they are nearly absent from the Magellanic clouds. Of the four objects in the polar squares, three are perhaps doubtful planetaries; the fourth is the "Owl" nebula.

The material here discussed does not support Professor Bohlin's conclusion that the planetary nebulæ have a somewhat uniform distribution over the celestial sphere.

§ 4. *The distribution of gaseous nebulæ other than planetary.*

The material for this discussion is in three classes:—

A. Nebulæ known to be gaseous, in two subdivisions:

(*a*) Small, and pretty well defined, such as M. 76 Persei (Roberts, vol. i. p. 37; Crossley Reflector Photographs, plate 5), which looks exceedingly like a spiral, but has a bright line spectrum. (Huggins, vol. ii. 127; Fath, *L.O.B.*, 149.)

These are plotted as circles in fig. 2.

(*b*) Larger, but not enormously extended; the Orion nebula or the nebula 30 Doradus are examples.

These are plotted by groups of 4 horizontal lines in fig. 2.

Both subdivisions of Class A are included in List III.

B. Nebulæ not known to be gaseous, but having the character-

istic appearance of gaseous nebulæ. These again are subdivided into:—

(*c*) Small and pretty well defined, such as the fan-shaped nebulæ near γ Cassiopeiæ. (Ind. Cat. 59.) Plotted as dots in fig. 2.

(*d*) Larger but not enormously extended nebulæ, such as the network N.G.C. 1499, photographed by Barnard and Roberts. Plotted by small groups of vertical lines in fig. 2.

Subdivisions (*c*) and (*e*) of Class B are included in List IV.

(*e*) Very extended nebulæ, such as those photographed by Barnard in Ophiuchus and Sagittarius, Max Wolf's "Cave" in Aquila, etc. Plotted by wide vertical lines in fig. 2. Centres from List V.

It will be understood that this material cannot be complete. I have included all the nebulæ known to be gaseous, and only those others of which photographs are available, or which are sufficiently bright to be recorded in Bailey's Catalogue of Bright Nebulæ and Clusters. Hence we cannot draw conclusions with the confidence that was legitimate in regard to the spirals and the planetary nebulæ. But provisionally we may say—

(5) that the somewhat extended gaseous nebulæ are more condensed upon the galaxy than are the planetaries.

(6) that their distribution in longitude is very unequal.

(7) that they are numerous in the greater Cloud of Magellan.

(8) but that they are not numerous in the galaxy in the square DG. Doubtless there are many gaseous nebulæ as yet unphotographed in this square; but inasmuch as Bailey's Catalogue gives some forty nebulæ in the cloud, and only two on the Milky Way itself, there is some reason to believe that the comparative rarity of gaseous or supposed gaseous nebulæ on the galaxy in square DG is a real phenomenon.*

§ 5. *The distribution of coarse clusters.*

The term "cluster" is necessarily vague, and there is no general understanding as to the distinction between coarse clusters and mere rich fields. In order to obtain a working standard I have used only those objects which are classified by Bailey as coarse clusters. They were plotted on the cubical projection, like the others; but I have not thought it necessary at present to reproduce the diagram, which can be described in few words:—

(9) The coarse clusters are, as is well known, strongly galactic. They avoid altogether the squares NG and SG.

(10) They are much scattered in BG and CG.

(11) They are scarce in AG.

*[Note added June 15:—The large nebula about λ Centauri, discovered by Frost at Arequipa, Harvard Nebulæ 788-91, and described by Innes and Wood, Transvaal Circular 4, has been omitted by an oversight from my figure.

I should also have included the remarkable annular or helical nebula N.G.C. 7293, photographed by Knox Shaw at Helwân, of which a lantern slide is in the collection of the Society.]

(12) But the few in AG and the great number in DG are strongly condensed on the galactic plane. In DG they make a narrow belt only a few degrees broad between two lines of planetary nebulæ—a very striking and curious arrangement.

(13) They are very strongly condensed in the greater Cloud of Magellan, where most of them are involved in nebulosity.

§ 6. *The distribution of globular clusters.*

The only difficulty in discussing the distribution of globular clusters arises from the fact that some of the objects described in the N.G.C. as globular clusters are really spiral nebulæ. Professor Bohlin has called attention to this (*K. Svenska Vet. Akad. Handlingar*, Bd. 43, No. 10, page 4), and gives as an instance—

N.G.C. 628 = M.74, which is described by Rosse, and photographed by Roberts (vol. ii. p. 67), and by Keeler (Crossley, plate 4), as a fine spiral, though in the N.G.C. it is described as a globular cluster.

Two other instances are N.G.C. 6221 and 6412, both of which are really spirals.

I have divided the N.G.C. globular clusters, excluding the three above, into two parts:—

(*a*) Those described by Bailey as globular clusters. Marked with a circle in figure (3).

(*b*) Those not in Bailey. Marked with a dot.

The distribution of these is very remarkable. The clusters in part (*a*) are contained very nearly in one hemisphere of the sky, whose pole is on the galactic plane in galactic longitude about 300°.

The heavy lines drawn in fig. 3 are a great circle about this pole. It will be seen at once that the globular clusters marked by circles trespass only a very few degrees beyond this circle; but one of the trespassers is the cluster in Hercules.

A few of the clusters in class (*b*) lie in the other hemisphere, in square BG. Two have already been removed from this square, as spiral nebulæ. It seems well to reserve judgment on the others till they have been photographed with high power.

Conclusions.

§ 7. The results of this reconnaissance may be summarised thus:—

	Avoid	Scattered in	Condensed in	In Magellanic Cloud.
Planetaries	NG; SG	BG; CG	AG; DG	No.
Other gaseous	NG; SG; DG	Clustered in longitude		Yes!!
Loose clusters	NG; SG; (AG)	BG; CG	(AG); DG	Yes.
Globular ,,	Avoid one hemisphere of the sky			Yes.

And the brief conclusion is, that statistical discussions with reference to galactic latitude only obscure the significant features of these distributions. They must be studied in detail upon charts.

I should lay emphasis upon the necessarily preliminary

character of these two papers on galactic distributions. They represent a first rough sorting of the material. Some attempt has been made to obtain a standard of importance by the use of Bailey's Catalogue of Bright Clusters and Nebulæ, but it has been impossible to avoid adding to our collection many objects not in Bailey. The least satisfactory part of my examination is probably the treatment of fairly extended gaseous nebulæ on the Milky Way in square DG. I shall be happy if this rough attempt to classify the gaseous nebulæ should suggest the publication of any evidence as to the character or spectrum of nebulæ, and especially of southern nebulæ, which may be lying unpublished.

List I.

N.G.C. and I.C. Numbers of Stellar Gaseous Nebulæ.

N.G.C.

5315	6565	6741	6533	6891
5873	6567	6766	6879	7027
6153	6578	6790	6881	
6439	6620	6803	6884	
6537	6644	6807	6886	

1st. Index Catalogue.

418	1266	1292	1297	

2nd. Index Catalogue.

1644	2448	4406	4642	4846
2149	2501	4544	4663	4997
2165	2553	4593	4699	5117
2189	2621	4634	4732	5217
2206	4191	4637	4776	

Three announced in H.C. 158.

	h m		° ′
R.A.	11 24·0	Decl.	−52 23
	16 29·6		−55 28
	17 32·0		−44 6

List II.

N.G.C. Numbers of Planetary Gaseous Nebulæ.

604	2438	3918	6337*	6894*
1032	2440	3982*	6369	6905
1501*	2792	4151	6543	7009
1514*	2818	5307	6563	7026
1535	2867	5882 (I.C. 1108)	6572	7354
2003*	3132	5979	6720	7662
2022	3195*	6049	6781	
2175*	3211	6210	6818	
2392	3242	6302	6826	
2415*	3587	6326	6853	

Index Cat. 1747. B.D. +30° 3639.

* Gaseous spectrum inferred; no determination known.

List III.

N.G.C. Numbers of Gaseous Nebulæ not Planetary.

650, 1	2023	6401
1698 = I.C. 2105	2029	6514
1714	2032	6523
1723 = I.C. 2111	2033 (neb. in clus.)	6618
(neb. in clus.)	2070	6751
1743	2079	6778
1748 = I.C. 2114	2080	6804
1763 = I.C. 2115	2086 = I.C. 2145	6960
1769 = I.C. 2116	4361	7008
1770 = I.C. 2117	5189 = I.C. 4274	7619
1935 = I.C. 2126	6175	
1936 = I.C. 2127	6309	

List IV.

N.G.C. Numbers of Nebulæ probably Gaseous, not exceedingly extended.

40	1727	1986	2081	6991
281	1755	2001	2083	6992, 5
346	1770	2002	2084	7023
395	1783	2006	2086	7129
456	1788	2018	2103	7293
460	1855	2020	2122	7822
465	1858	2030	2237	
707	1874	2040	2245	
1316*	1929	2050	2261	
1432, 5	1952	2058†	2264	
1499	1955	2064	2359	
1555	1962	2068, 71	3310	
1579	1968	2074	6611	
1722	1974	2077	6888	
Index Cat.				
59	2128	2169	5146	

B.D. + 57° 2309 involved in nebulosity.

New nebula in Monoceros (Roberts). Crossley 16. $7^h\ 0^m - 10°\ 18'$.

* Classification doubtful. Perhaps spiral.
† Classification doubtful. Perhaps globular cluster.

List V.

Approximate Centres of very extended Nebulosities.

R.A. h m	Dec. °	
4 13	+28	Barnard, *A.P.J.*, **25**, 220.
5 0	− 8	Wolf, *M.N.*, **65**, 528.
5 9	+34	Roberts.
		Wolf, *M.N.*, **63**, 506.
5 30	− 5	Orion nebula and extensions.
5 43	+ 1	Barnard, *A.P.J.*, **17**, 80.
10 33	−58	η Carinæ.
10 41	−59	
15 53	−26	π Scorpii, Barnard, *A.P.J.*, **23**, 144.
16 6	−19	ν Scorpii, Barnard, *A.P.J.*, **31**, 8.
16 20	−23	ρ Ophiuchi, Barnard, *A.P.J.*, **31**, 10.
17 20	−24	Barnard, *A.P.J.*, **9**, 19.
18 40	− 6	Barnard, *A.P.J.*, **2**, 352.
19 35	+10	Wolf, *Know.*, **25**, 203.
20 55	+44	Wolf, America nebula.
21 35	+57	Barnard, *A.P.J.*, **21**, 46.
22 10	+69	Wolf, *M.N.*, **69**, 117.

List VI.

N.G.C. Numbers of Clusters classed as Globular in Bailey's Catalogue.

104	2100	5986	6341	6712
330	2134	6093	6356	6715
362	2136	6121	6388	6723
376	2157	6171	6397	6752
1261	2164	6205	6402	6779
1291	2808	6218	6441	6809
1399	3201	6229	6541	6864
1711	4590	6254	6584	7078
1818	4833	6266	6624	7089
1850	5024	6273	6626	7099
1851	5139	6284	6637	
1904	5272	6293	6642	
2004	5286	6304	6656	
2031	5904	6333	6681	

List VI.—*continued.*

N.G.C. Numbers of Clusters classed as Globular, not in Bailey's Catalogue.

136	1697	3041	6101	6553
288	1782	3603	[6221]	6558
[628]	1854	4027	6256	6569
843	1902	4147	6287	6638
1310	2090	4372	6316	6934
1379	2098	5634	6362	6981
1387	2118	5897	[6412]	7099
1436	2298	5927	6522	7214
1512	2537	5946	6528	

Cambridge Observatory:
1911 *June* 9.

Observations of α Orionis by the late Joseph Baxendell, F.R.S.; together with the Magnitudes deduced from the Observations of Sir J. Herschel and of Argelander. Edited by H. C. Plummer, M.A.

1. The circumstances under which the following observations are now published may be briefly related. The known variability of α Orionis, which was discovered by Sir John Herschel,* was brought into general notice by the marked increase of brightness which occurred in the autumn of 1902. Bearing this in mind, I wished, during my stay at the Lick Observatory in 1907–8, to ascertain whether the radial velocity of the star was constant, or whether it showed changes associated with the changes in the luminosity of the star, as appears to be the case always with certain classes of variable stars. Facilities were kindly given by Dr. Campbell, and the results were published.† Changes in the radial velocity were quite clearly suggested, but with regard to their nature, and still more with regard to their period, nothing could with certainty be inferred.

Quite recently Herr Bottlinger ‡ has added other observations made at the Cape and at Potsdam, and from the combined material has deduced a period of six years, and finds the data consistent with the hypothesis that α Orionis is a spectroscopic binary. On the probability that this view is correct I can express no opinion. The changes apparent in the observed velocity are small, and the question is not altogether an easy one.

But the mere suggestion of a period of this kind makes it

* *Memoirs R.A.S.*, xi. p. 269.
† *Publ. Astr. Soc. Pac.*, x. p. 227.
‡ *A.N.*, 4467.

desirable to find whether a similar periodicity can be traced in the light-fluctuations of the star. A series of observations of α Orionis extending over more than forty years was made by the late Mr. Baxendell, and this unpublished material had been placed in the hands of Professor Turner, among a large amount of similar work on a considerable number of variables. The observations relating to α Orionis, extracted from Mr. Baxendell's ledgers, were most kindly put at my disposal by Professor Turner. They were made at Manchester until 1871, and at Southport from this date onwards.

2. The comparison stars used by Baxendell are few in number, and are given in the following list, with the designations here adopted for the sake of brevity, the magnitude taken from the Revised Harvard Photometry, and the spectral type.

Star.	Designation.	Mag.	Type.
α Aurigæ	A	0·21	G
β Orionis	B	0·34	B8p
α Can. Min.	C	0·48	F5
α Tauri	D	1·06	K5
α Orionis	X	...	Ma

Even of these, Capella occurs only in the first observation. Subsequent comparisons were rarely made with more than a single pair of stars, of which one was nearly always Aldebaran. Under these circumstances it is not possible to deduce from the observations any system of corrections to the magnitudes assigned above. A thorough discussion of the accuracy of the observations from internal evidence is thus out of the question; but there is no reason to doubt that the observer's skill and experience enabled him to reach the utmost accuracy attainable with the simple method adopted.

3. Up to the year 1854 the sequences (which are set down throughout in the order of decreasing brightness) are expressed in the notation * of Sir W. Herschel. As punctuation marks do not readily convey a quantitative meaning, numbers enclosed within brackets have been substituted according to the following scheme:—

"Simple Characters."

(0) . "Equality."
(1) , "The least perceptible difference more bright" (than).
(2) — "A very small difference more bright" (than).
(3) —, "A small difference more bright" (than).
(4) — — "A considerable difference more bright" (than).

"Compound Characters, expressing the Wavering of Star Light" between the above steps, are given in Herschel's notation

* *Phil. Trans.*, vol. 205, A, p. 400.

by writing the corresponding symbols one above the other. They are here expressed by writing the corresponding *numbers* in the same bracket. Thus, for example, (23) stands for [=,], and means a variation between (2) and (3). In later years the steps were expressed numerically by Baxendell himself, and are set down here (without brackets) with little change, except for the omission of decimal points which are unnecessary. Further slight variants occur here and there in the notation, and are now recorded in a form which should convey its meaning without ambiguity.

4. The method adopted to deduce the magnitude of α Orionis is extremely simple, yet it seems doubtful whether any essential advantage could be gained by following a more complicated procedure. The numbers expressing the steps, whether according to the earlier or later notations, have been treated as differences of magnitude on a scale indicated by the observation itself. In other words, when the variable is placed between two given stars, the magnitude is inferred by simple interpolation between the two assumed magnitudes according to the intervals denoted by the steps recorded. Thus, for example, the observation B(12) X(2)D leads to the equation

$$X = B + \frac{3}{3+4}(D - B) = 0^{m}34 + \frac{3}{7}(0^{m}72) = 0^{m}34 + 0^{m}31 = 0^{m}65.$$

It has been thought safer to make no attempt to determine the value of a step which could be used as a definite difference of magnitude over a series of dates. Thus, in general, it has been considered impossible to infer the magnitude of the variable unless α Orionis has been compared both with a brighter and a fainter star; and in those cases where a single comparison star has been observed, the observation is of value only in a qualitative sense.

5. The observations of Baxendell, reduced as explained above, are given in Table I. When this reduction had been made, it seemed natural to collect other observations before proceeding with the discussion of the variability of the star. In particular, the series could be prolonged, and certain gaps filled, by including the observations of Sir John Herschel * and of Argelander.† To make these series available, it was necessary to deduce from them the magnitudes of α Orionis according to the same scale, that of the Revised Harvard Photometry. Accordingly, they have been treated in the same way as Baxendell's observations, except for the fact that a wider choice of comparison stars was made by the two observers. The deduced magnitudes are given in Table II. for Sir John Herschel, and in Table III. for Argelander.

6. In the title of his paper, Sir John Herschel refers to the "Variability and Periodical Nature" of α Orionis. But although the fluctuations in the brightness of the star seem to have been unusually well marked during the period covered by his observa-

* *Memoirs R.A.S*, xi. p. 269.
† *Astr. Beob. zu Bonn*, vii. p. 491.

tions, he did not suggest a period, and α Orionis is now regarded as an irregular variable. On the other hand, Argelander * found a period of 196 days. Quite independently, from Baxendell's observations, I found a rather longer period of about 208 days. but the maximum variation in this period was less than ±0·1 magnitude, and this, even if proved to be real and permanent, goes very little way towards representing the observed changes. The longer period agrees with the view of Schmidt,† who in 1856 suspected that the period was perhaps 10 days longer than that assigned by Argelander, or that it might be variable.

But our more immediate purpose is to test the 6-year period found by Herr Bottlinger, as already mentioned. With this object I have used all the observations published in the present paper, and in addition a considerable number of later determinations extending to the year 1905. Unfortunately, in this later period there is no sustained series made by a practised observer available. In order to investigate at the same time a possible systematic error depending on the season, which might reasonably be suspected, the observations for each month have been treated separately. The monthly means have been set down in six columns, each column containing the means for every sixth year, beginning with the year 1836. The means for each column were then formed, and are now given in tabular form.

	1836 +6n.	1837 +6n.	1838 +6n.	1839 +6n.	1840 +6n.	1841 +6n.	Means.
Jan.	0·64	0·59	0·71	0·77	0·69	0·65	0·67
Feb.	0·67	0·51	0·74	0·72	0·73	0·74	0·68
Mar.	0·72	0·49	0·63	0·72	0·66	0·73	0·66
Apr.	0·73	0·60	0·65	0·80	0·67	0·84	0·71
Sept.	0·70	0·70	0·82	0·50	0·91	0·68	0·72
Oct.	0·65	0·55	0·77	0·49	0·62	0·68	0·63
Nov.	0·65	0·62	0·76	0·59	0·50	0·64	0·63
Dec.	0·62	0·73	0·65	0·67	0·56	0·64	0·65
Means	0·67	0·60	0·72	0·63	0·63	0·65	0·67

7. From the above figures and from an inspection of the observations, certain conclusions can now be drawn :—

(1) The mean magnitude of α Orionis between the years 1836 and 1905 was 0·67 on the Harvard scale.

(2) Extreme deviations of about ±0·4 mag. from this mean have been observed, but the times at which such deviations are to be expected are not yet within the possibility of prediction.

(3) The observations are practically free from any systematic error depending on the season of the year at which they are made.

* *Astr. Beob. zu Bonn*, vii. p. 388.
† *A.N.*, 1080.

(4) There is no significant variation of the magnitude with a period of 6 years.

No special investigation has been made of the effect of atmospheric absorption due to the altitude of the stars, and indeed the time of observation has been omitted in Table I. The precise time is not always clearly indicated in Baxendell's ledgers, and a satisfactory examination of the effect on this class of observations would have been difficult. It has been considered that an experienced observer like Baxendell would naturally bear this point in mind, and avoid large errors by choosing favourable times for observation. That this view is so far justified that errors of this kind may be treated as accidental seems to be sufficiently confirmed by the conclusion (3) above, and the evidence on which it is based.

TABLE I.

Baxendell's Observations.

Date.	J.D. (239)		Mag.	Date.	J.D. (239)		Mag.
1837.				1848.			
Jan. 10	2020	X=C+2, A=B+2	0·35	Nov. 13	6345	D(0) X	1·06
1840.				14	6346	D(0) X	1·06
Feb. 19	3155	X(1) D		17	6349	D(0) X	1·06
22	3158	X(1) D		18	6350	X(0) D	
24	3160	X(3) D				C(1) X(01) D	1·00
29	3165	X(01) D				C(12) X(0) D	
Mar. 3	3168	B(12) X(2) D	0·65	30	6362	D(0) X	1·06
16	3181	X(3) D		Dec. 1	6363	D(0) X	1·06
20	3185	X(3) D		2	6364	X(01) D	
1842.				13	6375	X(1) D	
Mar. 6	3901	X(3) D		14	6376	C(12) X(12) D	0·77
1846.				15	6377	X(2) D	
Jan. 20	5317	B(4) X(2) D	0·82	16	6378	C(1) X(12) D	0·71
28	5325	C(4) X(2) D	0·87	17	6379	C(12) X(2) D	0·73
1848.				20	6382	C(1) X(2) D	0·67
Sept. 19	6290	B(3) X(2) D	0·77	28	6390	C(2) X(2) D	0·77
Oct. 5	6306	B(4) X(2) D	0·82	1849.			
11	6312	X(2) D		Jan. 2	6395	X(23) D	
18	6319	B(3) X(2) D	0·77	3	6396	C(12) X(3) D	0·67
21	6322	B(3) X(2) D	0·77	11	6404	X(3) D	
Nov. 4	6336	B(34) X(2) D	0·80	14	6407	C(0) X(3) D	0·48
6	6338	B(4) X(0) D	1·06	15	6408	X(01) C, X(3) D	0·36
8	6340	X(01) D		30	6423	X(2) C, X (3) D	
12	6344	C(2) X(01) D	0·94	Feb. 8	6432	B(0) X(3) C	0·34

TABLE I.—*continued.*

Date.	J.D. (239)		Mag.
1849.			
Feb. 10	6434	X(3) C	
Mar. 6	6458	B(0) X(3) C	0·34
8	6460	B(01) X(23) C	0·36
24	6476	B(1) X(1) C	0·41
Apr. 3	6486	C(0) X(2) D	0·48
Sept. 5	6641	X(2) D	
18	6654	X(1) D	
22	6658	B(1) X(2) D	0·58
Oct. 10	6676	B(12) X(1) D	0·77
12	6678	B : X ≤ X : D	0·70
13	6679	B(2) X(2) D	0·70
15	6681	B(2) X(2) D	0·70
21	6687	B(1) X(1) C	0·41
28	6694	B(0) X(3) D	0·34
Nov. 4	6701	B(1) X(3) D	0·52
11	6708	B(2) X(3) D	0·63
16	6713	B : X < X : D	0·63
23	6720	B(2) X(2) D	0·70
27	6724	B(2) X(2) D	0·70
Dec. 3	6730	B(2) X(2) D	0·70
14	6741	B(2) X(2) D	0·70
22	6749	B(3) X(2) D	0·77
1850.			
Jan. 14	6772	C(2) X(12) D	0·81
26	6784	B(3) X(01) D	0·96
Feb. 1	6790	B(3) X(12) D C : X = X : D	0·80
3	6792	C : X < X : D	
Mar. 3	6820	X(0) C. X (23) D	0·48
12	6829	X(0) C, X(3) D	0·48
Sept. 13	7014	B(1) X(3) D	0·52
28	7029	B(2) X(3) D	0·63
Oct. 1	7032	B(2) X(23) D	0·66
9	7040	B(2) X(23) D	0·66
11	7042	B(2) X(3) D	0·63
21	7052	X(3) D	
28	7059	B(1) X(3) D	0·52
Nov. 2	7064	B : X < X : D	

Date.	J.D. (239)		Mag.
1850.			
Nov. 7	7069	B : X < X : D	
27	7089	B : X < X : D, X(1) C	
28	7090	B(2) X(3) D, C(0) X	0·55
Dec. 5	7097	B(1) X(3) D	0·52
6	7098	X(0) C	0·48
9	7101	X(01) C	
17	7109	B(1) X(3) D	0·52
1851.			
Jan. 5	7128	B : X = X : D	0·70
Feb. 1	7155	B(3) X(2) D	0·77
23	7177	B(2) X(2) D C(1) X(2) D	0·68
Mar. 1	7183	C(2) X(3) D	0·71
20	7202	C : X ≥ X : D	0·77
Apr. 7	7220	C(2) X(01) D	0·94
Nov. 2	7429	B(2) X(3) D	0·63
Dec 3	7460	B(01) X(1) C	0·39
1852.			
Feb. 18	7537	B(2) X(3) D	0·63
24	7543	C(1) X(3) D	0·63
Mar. 3	7551	X(01) C	
Apr. 3	7582	C : X ≤ X : D	0·77
1853.			
Nov. 1	8159	B : X = X : D	0·70
1854.			
Oct. 31	8523	B3 X2 D	0·77
Nov. 3	8526	B : X ≥ X : D	0·70
8	8531	B2 X1 D	0·82
15	8538	B2½ X1 D	0·85
Dec. 9	8562	B2½ X1 D	0·85
18	8571	B2 X1 D	0·82
27	8580	B : X = X : D	0·70
1855.			
Feb. 9	8624	X(0) B	0·34
Oct. 23	8880	X(3) D	
27	8884	X(0) B	0·34
31	8888	B(0) X	0·34
Nov. 3	8891	B(01) X	

TABLE I.—*continued.*

Date.	J.D.		Mag.
1855.	(239)		
Nov. 10	8898	B(01) X(3) D	0·44
16	8904	B(01) X(3) D	0·44
20	8908	B(0) X(3) D	0·34
24	8912	B(1) X(3) D	0·52
27	8915	B1 X2 D	0·58
29	8917	B1 X1 D	0·70
Dec. 5	8923	B1 X2 D	0·58
6	8924	B1 X2 D	0·58
12	8930	B1 X2 D	0·58
18	8936	B1 X2 D	0·58
19	8937	B1 X3 D	0·52
20	8938	B1 X3 D	0·52
21	8939	B1 X2 D	0·58
29	8947	B1 X1½ D	0·63
1856.			
Jan. 9	8958	B1 X2 D	0·58
10	8959	B1 X1½ D	0·63
15	8964	B1 X1½ D	0·63
Feb. 2	8982	B(1) X(2) D	0·58
Oct. 19	9242	B2½ X1 D	0·85
Nov. 6	9260	X2 D	
8	9262	B1 X3 D	0·52
9	9263	B1 X3 D	0·52
10	9264	B1 X3 D	0·52
12	9266	B1 X3 D	0·52
27	9281	B1 X3 D	0·52
29	9283	B1 X2½ D	0·55
30	9284	B1 X3 D	0·52
Dec. 13	9297	B1 X2½ D	0·55
14	9298	B1 X2 D	0·58
28	9312	B1 X2 D	0·58
1857.			
Jan. 6	9321	B1 X3 D	0·52
12	9327	B1 X2 D	0·58
13	9328	B1 X3 D	0·52
19	9334	B1 X3 D	0·52
21	9336	B1 X2½ D	0·55
23	9338	B1 X3½ D	0·50
1857.	(239)		
Jan. 28	9343	B1 X3 D	0·52
Feb. 11	9357	B=X1½ C	0·34
Mar. 8	9382	B1 X1¼ D	0·66
9	9383	B1 X1¼ D	0·66
25	9399	C1 X2 D	0·67
Oct. 28	9616	B=X2 D	0·34
Nov. 15	9634	B1 X2½ D	0·55
18	9637	B1 X2½ D	0·55
23	9642	B1½ X2 D	0·65
26	9645	B1 X2 D	0·58
27	9646	B1¼ X2 D	0·61
Dec. 4	9653	B1 X1 D	0·70
7	9656	B1 X2 D	0·58
20	9669	B1 X2 D	0·58
24	9673	B=X3 D	0·34
31	9680	B½ X2½ D	0·46
1858.			
Jan. 4	9684	B½ X2 D	0·48
16	9696	B1 X2 D	0·58
20	9700	B1 X3 D	0·52
Feb. 14	9725	B½ X2¾ D	0·45
19	9730	B½ X3 D	0·44
Mar. 11	9750	C1 X2¾ D	0·64
25	9764	C1 X2 D	0·67
Apr. 13	9783	X>C, X5 D	
17	9787	C½ X4 D	0·54
Oct. 28	9981	D≥X	
Nov. 2	9986	B3½ X½ D	0·97
10	9994	B4 X=D	1·06
12	9996	B4½ X=D	1·06
15	9999	B4½ X½ D	0·99
	(240)		
17	0001	B4 X¾ D	0·95
18	0002	B3½ X1 D	0·90
21	0005	B3 X1 D	0·88
Dec. 2	0016	B2½ X1½ D	0·79
18	0032	B3 X2 D	0·77
21	0035	B2 X1 D	0·82

TABLE I.—*continued.*

Date.	J.D.		Mag.
1859.	(240)		
Jan. 6	0051	B3½ X2 D	0·80
22	0067	B1½ X3½ D	0·56
Oct. 21	0339	X3 D	
26	0344	B2½ X1½ D	0·79
29	0347	B4½ X=D	1·06
Nov. 20	0369	X1 D	
1860.			
Jan. 6	0416	B4 X1 D	0·92
11	0421	X=D	1·06
15	0425	B3½ X1 D	0·90
25	0435	B3 X1 D	0·88
27	0437	B3 X½ D	0·96
Feb. 6	0447	B3½ X½ D	0·97
9	0450	B4½ X=D	1·06
12	0453	D½ X	1·10
13	0454	B4½ X=D	1·06
16	0457	D½ X	1·10
19	0460	D½ X	1·10
26	0467	X=D	1·06
28	0469	X½ D	
Mar. 21	0491	X2 D	
Sept. 27	0681	B2½ X2½ D	0·70
Oct. 11	0695	B1 X2½ D	0·55
1861.			
Jan. 1	0777	B2½ X1½ D	0·79
4	0780	B2 X1½ D	0·75
Feb. 9	0816	B3 X1½ D	0·82
Mar. 8	0843	X2½ D	
30	0865	C=X5 D	0·48
Apr. 11	0877	X5½ D	
Oct. 27	1076	B=X4 D	0·34
Dec. 3	1113	B=X3½ D	0·34
25	1135	B1 X3 D	0·52
1862.			
Jan. 5	1146	B1 X3 D	0·52
Nov. 15	1460	B2 X3 D	0·63
Dec. 30	1505	B=X3 D	0·34
1863.			
Jan. 23	1529	B2½ X3 D	0·67
Oct. 30	1809	B3 X=D	1·06
Nov. 2	1812	B3 X½ D	0·96
4	1814	B3 X1 D	0·88
8	1818	B3 X1 D	0·88
29	1839	B4 X=D	1·06
Dec. 2	1842	B3 X1 D	0 88
5	1845	B3½ X=D	1·06
17	1857	B3½ X1 D	0·90
1864.			
Feb. 8	1910	B2 X1 D	0·82
Oct. 7	2152	B4 X1 D	0·92
9	2154	B4 X1 D	0·92
31	2176	B4 X1 D	0·92
Dec. 21	2227	B3 X1 D	0·88
31	2237	B3 X1 D	0·88
1865.			
Jan. 27	2264	B3 X1 D	0·88
Feb. 8	2276	X1 D	
Oct. 19	2529	B>X2½ D	
Nov. 21	2562	B2 X3 D	0·63
Dec. 22	2593	B3 X1 D	0·88
1866.			
Feb. 3	2636	B3 X2 D	0·77
17	2650	B=X4 D	0·34
Mar. 13	2674	B=X3½ D	0·34
18	2679	B1 X3 D	0·52
Oct. 6	2881	B1 X2 D	0·58
30	2905	B2 X2 D	0·70
Nov. 13	2919	B2 X1½ D	0·75
16	2922	B2 X1½ D	0·75
30	2936	B1½ X2 D	0·65
Dec. 7	2943	B½ X2½ D	0·46
14	2950	B½ X2½ D	0·46
30	2966	B½ X2½ D	0·46
31	2967	B1 X2½ D	0·55
1867.			
Jan. 8	2975	B½ X2½ D	0·46

TABLE I.—*continued.*

Date.	J.D. (240)		Mag.
1867.			
Jan. 28	2995	B1 X2⅓ D	0·55
Feb. 24	3022	B1 X2½ D	0·55
Oct. 28	3268	B1 X2 D	0·58
Nov. 5	3276	B2 X1½ D	0·75
26	3297	B2 X1½ D	0·75
Dec. 17	3318	B2 X2 D	0·70
1868.			
Jan. 3	3335	B1½ X2 D	0·65
17	3349	B1 X2½ D	0·55
26	3358	B1 X1½ D	0·63
Mar. 27	3419	X>C, X2¼ D	
Nov. 8	3645	B1 X3 D	0·52
1869.			
Feb. 6	3735	B2 X3 D	0·63
Mar. 2	3759	B1 X2½ D	0·55
Nov. 10	4012	B2 X1 D	0·82
Dec. 22	4054	B2 X2 D	0·70
1870.			
Jan. 23	4086	B1 X2 D	0·58
1871.			
Mar. 16	4503	B=X2 D, C=X	0·41
1873.			
Jan. 20	5179	B3 X1 D	0·88
28	5187	C2½ X½ D	0·96
Dec. 13	5506	B2½ X=D	1·06
Dec. 20	5513	B2 X1 D	0·82
1876.			
Jan. 21	6275	B2½ X½ D	0·94
1877.			
Nov. 24	6948	B1½ X2 D	0·65
1878.			
Jan. 25	7010	B1 X1½ D	0·63
31	7016	C½ X1¾ D	0·61
Oct. 27	7285	X3 D	
1880.			
Feb. 21	7767	D>X	
26	7772	D>X	
Mar. 8	7783	C3 X=D	10·6
1881.			
Jan. 21	8102	C4 X2 D	0·87
Feb. 20	8132	C4 X½ D	1·00
23	8135	C5 X=D	1·06
Dec. 10	8425	B7 X, D1 X	1·18
1882.			
Feb. 15	8492	C3 X2 D	0·83
1883.			
Jan. 30	8841	C3 X2 D, B4 X	0 83
Feb. 23	8865	C2 X3 D	0·71
1884.			
Nov. 27	9508	B1 X4½ D	0·47

TABLE II.

Magnitudes deduced from Sir John Herschel's Observations.

Date.	J.D. (239)	Mag.
1836.		
Mar. 22	1726	1·20
Nov. 12	1961	0·34
13	1962	0·34
26	1975	0·47
1837.		
Dec. 29	2373	0·77
1838.		
Jan. 2	2377	0·90
6	2381	0·83
Jan. 13	2388	0·83
Feb. 25	2431	0·77
Apr. 14	2479	0·70
1839.		
Jan. 16	2756	1·20
17	2757	1·14
22	2762	1·14
Nov. 26	3070	0·30
30	3074	0·48
Dec. 11	3085	0·41
29	3103	0·77
1840.		
Jan. 2	3107	0·67
5	3110	0·67
6	3111	0·77
7	3112	0·77
Feb. 25	3161	0·87
Apr. 18	3214	0·67

TABLE III.

Magnitudes deduced from Argelander's Observations.

Date.	J.D. (239)	Mag.	Date.	J.D. (239)	Mag.	Date.	J.D. (239)	Mag.
1842.			1843.			1844.		
Oct. 8	4117	1·20	Mar. 20	4280	0·44	Apr. 10	4667	0·43
17	4126	0·65	Apr. 16	4307	0·71	19	4676	0·63
21	4130	0·72	Oct. 2	4476	0·88	26	4683	0·71
28	4137	0·82	22	4496	0·82	28	4685	0·73
Nov. 2	4142	0·74	27	4501	0·81	Oct. 31	4871	0·71
5	4145	0·82	Nov. 3	4508	0·77	Nov. 7	4878	0·56
9	4149	0·92	4	4509	0·75	Dec. 4	4905	0·66
10	4150	0·86	12	4517	0·76	5	4906	0·58
13	4153	0·92	13	4518	0·77	6	4907	0·64
17	4157	0·88	20	4525	0·82	8	4909	0·68
21	4161	1·16	Dec. 10	4545	0·82	11	4912	0·67
26	4166	1·16	16	4551	0·98	21	4922	0·72
28	4168	1·10	1844.			1845.		
Dec. 1	4171	0·97	Jan. 9	4575	1·15	Jan. 28	4960	0·58
2	4172	0·90	11	4577	1·05	Feb. 8	4971	0·66
3	4173	1·03	12	4578	1·06	Oct. 24	5229	0·26
5	4175	1·00	13	4579	1·05	29	5234	0·30
13	4183	0·74	15	4581	1·10	Nov. 3	5239	0·30
14	4184	0·71	23	4589	1·14	5	5241	0·30
15	4185	0·80	Feb. 6	4603	0·97	7	5243	0·30
22	4192	0·85	12	4609	0·83	22	5258	0·34
26	4196	0·80	13	4610	0·85	28	5264	0·40
1843.			14	4611	0·78	30	5266	0·37
Jan. 7	4208	0·65	15	4612	0·79	1846.		
8	4209	0·55	19	4616	0·71	Jan. 13	5310	0·70
10	4211	0·56	27	4624	0·56	15	5312	0·70
13	4214	0·62	Mar. 3	4629	0·58	Feb. 21	5349	0·74
20	4221	0·74	6	4632	0·48	22	5350	0·87
21	4222	0·66	14	4640	0·19	28	5356	0·81
22	4223	0·70	29	4655	0·34	Apr. 9	5396	0·40
Feb. 7	4239	0·59	30	4656	0·43	Oct. 13	5583	0·30
12	4244	0·51	Apr. 1	4658	0·39	23	5593	0·30
13	4245	0·42	2	4659	0·34	Nov. 2	5603	0·30
22	4254	0·60	4	4661	0·36	10	5611	0·40
Mar. 2	4262	0·63	5	4662	0·39	11	5612	0·32
7	4267	0·61	7	4664	0·34	16	5617	0·34
17	4277	0·34	8	4665	0·36	27	5628	0·30

TABLE III.—*continued.*

Date.	J.D. (239)	Mag.
1846.		
Dec. 13	5644	0·34
17	5648	0·39
1847.		
Jan. 10	5672	0·58
13	5675	0·57
25	5687	0·39
Feb. 23	5716	0·80
Mar. 1	5722	0·68
5	5726	0·77
15	5736	0·67
16	5737	0·77
17	5738	0·74
Sept. 18	5923	0·30
Oct. 15	5950	0·34
Nov. 5	5971	0·30
10	5976	0·30
Dec. 5	6001	0·30
25	6021	0·44
1848.		
Jan. 3	6030	0·56
9	6036	0·51
26	6053	0·48
Feb. 1	6059	0·53
27	6085	0·70
Mar. 7	6094	0·74
11	6098	0·76
21	6108	0·83
27	6114	0·80
31	6118	0·82
Apr. 1	6119	0·77
2	6120	0·75
17	6135	0·73
Sept. 25	6296	0·63
Oct. 2	6303	0·61
3	6304	0·70
6	6307	0·85
7	6308	0·58
22	6323	0·65

Date.	J.D. (239)	Mag.
1848.		
Oct. 23	6324	0·73
26	6327	0·71
Nov. 25	6357	0·75
Dec. 18	6380	0·57
23	6385	0·62
1849.		
Jan. 15	6408	0·38
Feb. 11	6435	0·37
23	6447	0·64
Mar. 20	6472	0·58
28	6480	0·60
Apr. 4	6487	0·40
Oct. 21	6687	0·57
1851.		
Jan. 8	7131	0·58
21	7144	0·59
23	7146	0·63
30	7153	0·70
Feb. 14	7168	0·69
Mar. 2	7184	0·81
24	7206	0·57
28	7210	0·70
Oct. 13	7409	0·28
20	7416	0·29
Nov. 1	7428	0·25
23	7450	0·31
Dec. 12	7469	0·21
19	7476	0·33
25	7482	0·34
1852.		
Jan. 10	7498	0·53
12	7500	0·61
26	7514	0·67
Feb. 24	7543	0·57
Mar. 14	7562	0·65
26	7574	0·73
Apr. 2	7581	0·58
16	7595	0·69

Date.	J.D. (239)	Mag.
1852.		
Apr. 21	7600	0·72
Oct. 12	7774	0·30
17	7779	0·27
20	7782	0·25
Nov. 6	7799	0·28
18	7811	0·28
Dec. 8	7831	0·30
1853.		
Jan. 27	7881	0·34
Feb. 28	7913	0·63
Mar. 27	7940	0·98
28	7941	0·94
29	7942	0·87
Apr. 1	7945	0·84
7	7951	0·94
Oct. 24	8151	0·82
31	8158	0·70
Nov. 12	8170	0·60
30	8188	0·76
Dec. 2	8190	0·77
3	8191	0·72
24	8212	0·62
1854.		
Jan. 18	8237	0·55
21	8240	0·63
26	8245	0·64
Feb. 14	8264	0·55
Mar. 16	8294	0·48
17	8295	0·70
19	8297	0·71
Apr. 11	8320	0·56
17	8326	0·64
Sept. 27	8489	0·55
28	8490	0·51
Oct. 26	8518	0·43
30	8522	0·70
Dec. 26	8579	0·57

TABLE III.—*continued.*

Date.	J.D. (239)	Mag.
1855.		
Mar. 5	8648	0·34
11	8654	0·45
16	8659	0·41
Apr. 6	8680	0·39
11	8685	0·48
16	8690	0·40
Oct. 20	8877	0·50
1856.		
Jan. 13	8962	0·53
14	8963	0·61
Feb. 3	8983	0·79
4	8984	0·74
16	8996	0·66
Mar. 7	9016	0·68
11	9020	0·79
16	9025	0·89
24	9033	0·72
27	9036	0·77
28	9037	0·81
Apr. 5	9045	0·77
16	9056	0·71
19	9059	0·63
20	9060	0·73
Oct. 4	9227	0·85
20	9243	0·70
25	9248	0·67
27	9250	0·72
29	9252	0·73
Nov. 2	9256	0·77
Dec. 15	9299	0·55
16	9300	0·51
17	9301	0·60
24	9308	0·50
29	9313	0·49
1857.		
Jan. 2	9317	0·47
17	9332	0·71
20	9335	0·55
30	9345	0·66
Feb. 5	9351	0·63
8	9354	0·62
12	9358	0·62
14	9360	0·74
15	9361	0·72
20	9366	0·67
22	9368	0·70
24	9370	0·77
Mar. 3	9377	0·73
16	9390	0·67
17	9391	0·63
20	9394	0·67
26	9400	0·77
29	9403	0·68
Apr. 1	9406	0·77
Sept. 27	9585	0·29
Oct. 15	9603	0·26
17	9605	0·24
22	9610	0·34
23	9611	0·37
27	9615	0·48
Nov. 11	9630	0·52
15	9634	0·68
20	9639	0·62
Dec. 4	9653	0·70
14	9663	0·58
18	9667	0·61
1858.		
Jan. 3	9683	0·42
21	9701	0·30
22	9702	0·30
26	9706	0·29
Feb. 7	9718	0·28
17	9728	0·47
21	9732	0·43
26	9737	0·65
Mar. 19	9758	0·64
25	9764	0·71
Mar. 28	9767	0·68
29	9768	0·70
30	9769	0·68
Apr. 14	9784	0·72
Oct. 8	9961	0·52
12	9965	0·61
15	9968	0·68
30	9983	0·68
31	9984	0·60
Nov. 3	9987	0·77
9	9993	0·85
	(240)	
23	0007	1·00
30	0014	0·88
1859.		
Jan. 1	0046	0·87
5	0050	0·95
9	0054	0·84
20	0065	0·75
21	0066	0·83
Feb. 11	0087	0·84
Mar. 2	0106	0·67
7	0111	0·66
8	0112	0·66
20	0124	0·61
Apr. 1	0136	0·57
Nov. 3	0352	0·77
6	0355	0·70
8	0357	0·75
12	0361	0·76
18	0367	0·85
22	0371	0·77
1861.		
Nov. 9	1089	0·29
1864.		
Feb. 6	1908	0·89
Mar. 8	1939	0·77
9	1940	0·69
Apr. 8	1970	0·87

Observations of Saturn's Ninth Satellite (Phœbe) made at the Royal Observatory, Greenwich, in 1910.

(*Communicated by the Astronomer Royal.*)

The ninth satellite of Saturn was under observation near the opposition of 1910 October, and photographs were obtained with the 30-inch reflector whenever possible, but, owing to the unfavourable weather, successful photographs were secured on five nights only.

The position of the satellite has been measured on the photographs of September 27 and 28 with reference to four faint stars whose positions were determined from an astrographic plate of the region, as described in former communications. It was unfortunately not possible to get an astrographic plate of the region covering the positions Nov. 1–7, owing to cloudy weather and pressure of other work. The positions on these nights were therefore referred to all the stars shown on each plate whose positions were given in the catalogues of the Astronomische Gesellschaft.

To determine the position of the satellite relatively to Saturn, the right ascensions and declinations of the planet given in the *Nautical Almanac* have been corrected as shown in the latter part of this paper.

The G.M.T. is the arithmetical mean of the times of beginning and end of exposure.

Observations of Phœbe.

Date and G.M.T.	Exp.	Apparent R.A.	Apparent Dec.	Phœbe – Saturn. R.A.	Phœbe – Saturn. Dec.
d h m	m	h m s	° ′ ″	m s	′ ″
Sept. 27 15 8·6	45	2 13 48·46	+10 31 28·7	−1 48·59	−12 5·5
28 12 21·7	90	2 13 35·84	10 30 15·6	1 48·60	12 5·3
Nov. 1 9 50·1	90	2 4 8·32	9 38 39·3	1 30·81	10 44·8
1 11 28·8	90	2 4 7·16	9 38 32·9	1 30·71	10 44·7
2 9 22·2	90	2 3 51·35	9 37 12·3	1 29·81	10 39·4
2 10 57·9	90	2 3 50·04	9 37 5·1	1 29·91	10 40·5
7 10 18·7	90	2 2 25·01	9 29 57·6	1 25·02	10 10·6

Errors of the Tabular Place of Saturn.—To eliminate the error of the tabular place of Saturn (for comparison with the R.A. and Dec. of the satellite deduced as above), and also that arising from any systematic error of the catalogues employed, photographs of Saturn were taken with the 26-inch refractor, using the occulting shutter. By this means good measurable images of Saturn were obtained, together with sufficiently exposed images of the reference stars. Four images of Saturn and of six

stars were measured on each plate, the positions of the stars being deduced from measures made on an astrographic reference plate, using all the stars on the plate whose places were given in the Astronomische Gesellschaft Catalogue. The observed positions of Saturn are thus affected by any systematic errors of the catalogue, and are comparable with the positions of the satellite deduced with reference to the same stars. The following are the results obtained :—

Errors of N.A. Tabular Places of Saturn.

1910.	Tab. − Obs. R.A.	Dec.
	s	"
Dec. 6	−0·18	−0·4
6	−0·17	+0·1
Mean	−0·18	−0·2

The following corrections have therefore been applied to Saturn's tabular place :—

R.A. $+0^{s}\cdot18$ Dec. $+0''\cdot2$

Royal Observatory, Greenwich :
1911 *June.*

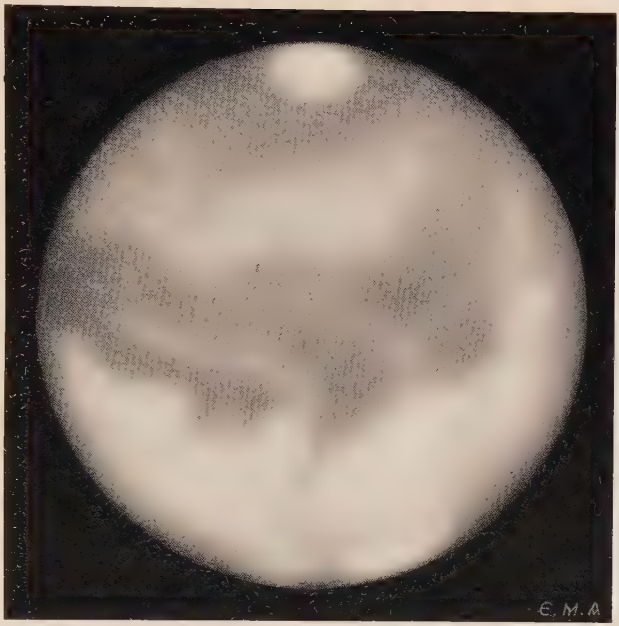
On some Drawings from Photographs of Mars taken in 1909 *by Professor Barnard and Professor Hale.* By E. M. Antoniadi. (Plates 17, 18.)

The two plates accompanying this paper show all the topographic details of Mars which the writer could find with certainty on some photographs taken in 1909 by Professor Barnard with the 40-inch Yerkes refractor and by Professor Hale with the 60-inch Ritchey mirror on Mount Wilson.

These photographs are the finest that have ever been made of the planet, revealing, as they do, more delicate objective detail than any ever drawn prior to 1909. It would be too long to give an exhaustive account of the markings recorded on them ; yet particular mention should be made of (1) the altered form of Syrtis Major ; (2) the two dusky "bridges" of the same to N. ; (3) the vast size of Lacus Mœris ; (4) the shading of Libya ; (5) the "promontory" to W. of Syrtis Major ; (6) the triple structure of Deltoton Sinus ; (7) the curved bright marking off Hammonis Cornu : (8) the great breadth of the Forked Bay ; (9) the shading of Hesperia ; (10) the complex, irregular "lake" on Hesperia, seen as a dark straight line by Schiaparelli in 1894 ;* (11) the pointed

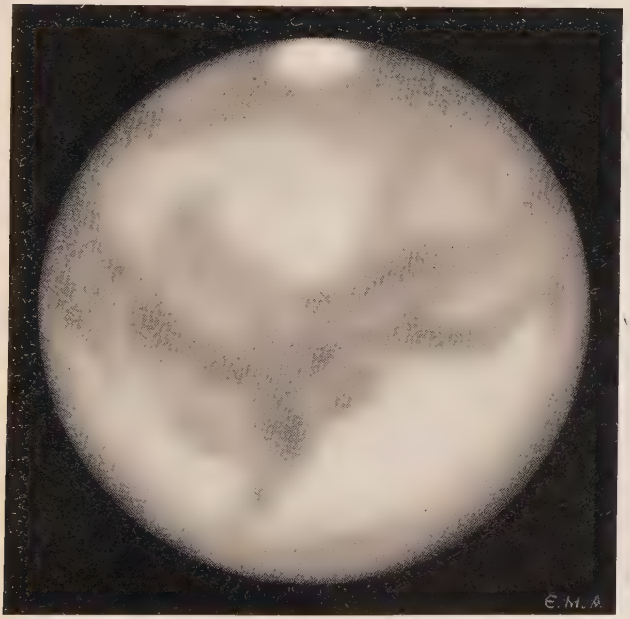
* *Astr. Nachr.*, No. 3271.

FIG. 1.



1909 SEPTEMBER 24. $\omega=17^{\circ}$. $\phi=-20^{\circ}\cdot4$.

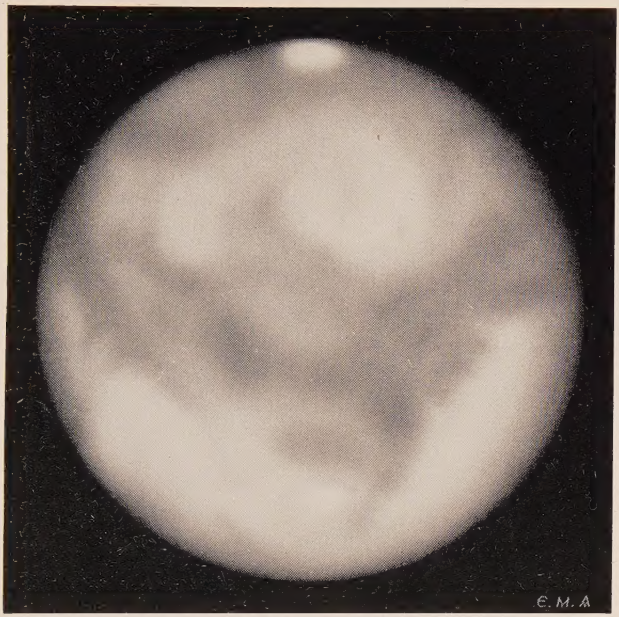
FIG. 2.



1909 SEPTEMBER 28. $\omega=295^{\circ}\pm$. $\phi=-20^{\circ}\cdot7$.

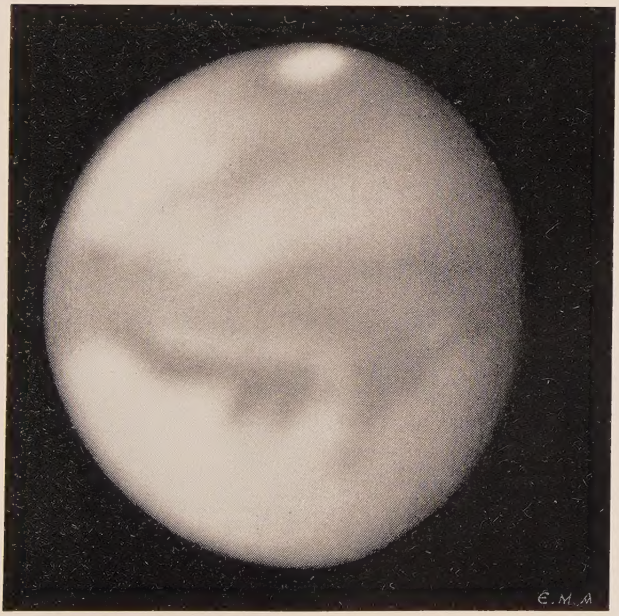
DRAWINGS FROM PHOTOGRAPHS OF MARS TAKEN BY PROF. E. E. BARNARD.

FIG. 3.



1909 OCTOBER 5. $\omega = 271^{\circ}\pm$. $\phi = -21^{\circ}.3$.

FIG. 4.



1909 NOVEMBER 3. $\omega = 12^{\circ}\pm$. $\phi = -23^{\circ}.4$.

DRAWINGS FROM PHOTOGRAPHS OF MARS TAKEN BY PROF. G. E. HALE.

form of Mare Cimmerium to W.; and (12) the double "bay" at the Cyclops "estuary." All these peculiarities were pointed out by the writer before the receipt of these photographs,* as they were quite clear to him in the 32·7-inch double equatorial of the Meudon Observatory, which he was enabled to use through the kindness of Dr. Deslandres.

Among other points of general interest revealed by these photographs may be mentioned:—

(*a*) The duskiness of the limb of Mars. In a note published in the *Monthly Notices* for December 1908, attention was called to the fact that, on Professor Lowell's 1907 photographs, the limb of the planet was not brighter than the central regions of the disc. But the photographs by Professor Barnard, taken in yellow light, and those of Professor Hale, obtained with a red screen, show the limb actually darker than the centre, as if the atmosphere of Mars, like that of the major planets, exerted some absorption on the rays reflected from its surface. Thus the bright limb, which is so striking a feature in visual work, seems due to contrast; and this conclusion is countenanced by the fact that the limb is not brighter when Mars is photographed without coloured screens.†

(*b*) The photographs show a great variety of shadings in the "continental" regions, which are thus seen not to be uniform yellow expanses, as sometimes represented.

(*c*) The bulging form of Syrtis Major to E. was photographed in 1905 and 1907 by Professor Lowell, but it does not exist on the 1909 photographs. This fact demonstrates that a vast tract of dusky material became yellow in appearance between the last two apparitions, the change being a return to a former state.‡

(*d*) On the other hand, the bright curved marking emerging out of Hammonis Cornu to N.E. is faintly shown by Green in 1877.§ Hence the inference that a remarkable stability characterises at least some of the minute features of the Martian surface, since an interval of thirty-two years did not affect the appearance of the same marking.

(*e*) Hellespontus, usually a dark streak, is scarcely recognisable on the 1909 photographs. Probably it was veiled by haze at the time. To this circumstance should be attributed also the absence of faint, irregular streaks from several of the "continental" regions during the last apparition.

A word or two on the so-called "canals" may not be out of place here. Using a $9\frac{3}{4}$-inch, on 1896 June 12, the writer glimpsed the Læstrygon as a straight black line, for a quarter of a second, on a disc of $6''\cdot7$; but on 1909 November 5, with the 32·7-inch, on a disc of $17''\cdot0$, he held steadily the sinuous curves and knots of the

* Letters to Schiaparelli (*Rivista di Astronomia*, 1909); *Astr. Nachr.*, No. 4359; *Bulletin S.A.F.*, November 1909 and January 1910; *Rivista di Astronomia*, December 1909, etc.

† As on M. Quénisset's photographs of 1909.

‡ To the outline drawn by Dawes in 1864.

§ *Mem. R.A.S.*, vol. xliv., pl. ii. fig. 11.

same streak. Again, on 1878 February 28, Schiaparelli glimpsed, with an $8\frac{1}{2}$-inch, the Hiddekel as a straight line, on a disc of $5''{\cdot}5$; but on 1909 November 3, on a disc of $17''{\cdot}4$, Professor Hale saw this streak, with the large mirror, as a broad, curved marking, and further resolved it into "minute, curved, and twisted filaments." Considering the distance of Mars in these cases, and the separating powers used, we find, for the Læstrygon, that Mars was seen at Meudon on 1909 November 5, as far as definition is concerned, as if it were almost nine times nearer the Earth than at Juvisy on 1896 June 12; and that it appeared, for the Hiddekel, on 1909 November 3, at Mount Wilson, as if it were some fifteen times nearer the Earth than at Milan on 1878 February 28. But it is a truth which nobody will question, that a celestial body is better seen when near the Earth than when far removed from it. Hence the fact that irregular streaks held steadily in perihelion opposition are glimpsed as straight lines when Mars is practically nine or fifteen times more distant establishes the illusive character of the straight "canals" as objective realities on the planet, and demonstrates the perfectly natural appearance of Mars.

Paris:
1911 *June* 19.

Errata in Professor Brown's paper "On a New Family of Periodic Orbits," etc.

Page 439, the first of equations (4), *for* "$-2nr\theta$" *read* "$-2nr\dot{\theta}$".

„ „ „ second „ „ "$\frac{\partial \Omega}{\partial \dot{\theta}}$" *read* "$\frac{\partial \Omega}{\partial \theta}$".

„ 442, last line, *for* "$r =$" *read* "$\ddot{r} =$".

„ 443, line 13, *for* "$\partial \Omega \hat{\imath} / \partial r$" *read* "$\partial \Omega_1 / \partial r$".

„ „ „ 8 from foot, *for* "$-\theta \frac{1}{r} \frac{\partial^2 \Omega}{\partial r \partial \theta}$" *read* "$-\dot{\theta} \frac{1}{r} \frac{\partial^2 \Omega}{\partial r \partial \theta}$".

„ „ „ 3 „ „ "$r(\theta + n)^2$" „ "$r(\dot{\theta} + n)^2$".

The erratum on page 603 should read—

p. 391, line 6 from bottom, in the denominators, *for* (κ) *read* $\sqrt{1 - \phi^2}$.